Systèmes à événements discrets à paramètres temporels

Olivier H. Roux

École Centrale de Nantes / IRCCyN

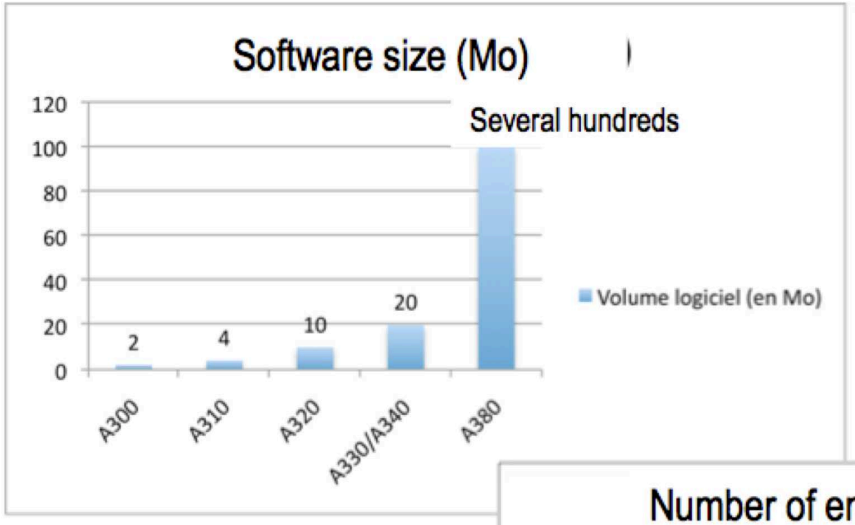
Modélisation des Systèmes Réactifs (MSR'13)

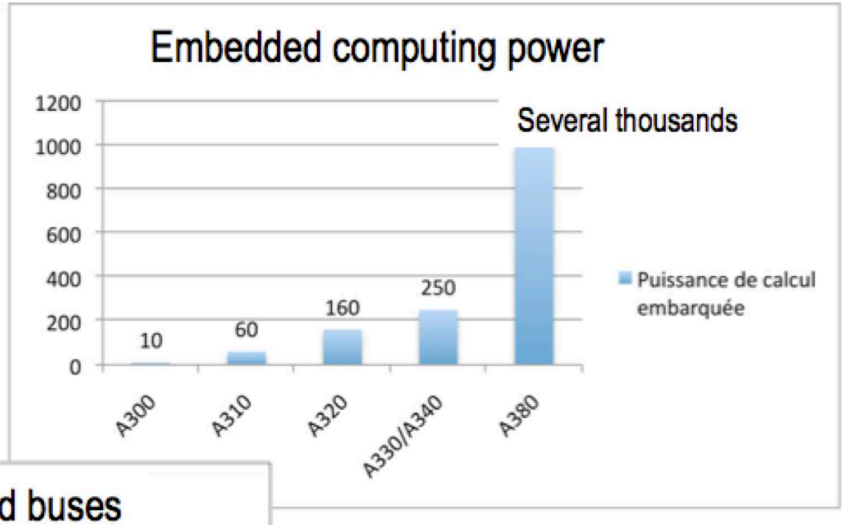
Rennes, 13 novembre 2013

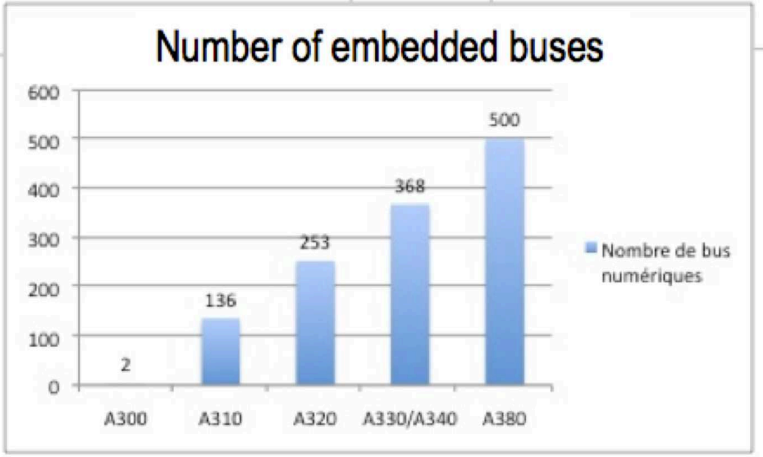
Avant propos

Résultats issus de travaux effectués avec :

- Aleksandra Jovanović
- Didier Lime
- Claude Martinez
- Louis-Marie Traonouez

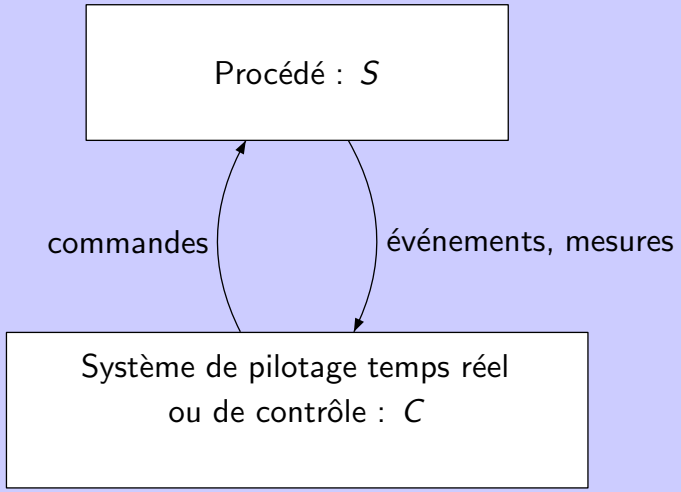






- Exponential increase of embedded systems in the aircraft
  - Computers, buses, middleware, software

Systèmes Réactifs



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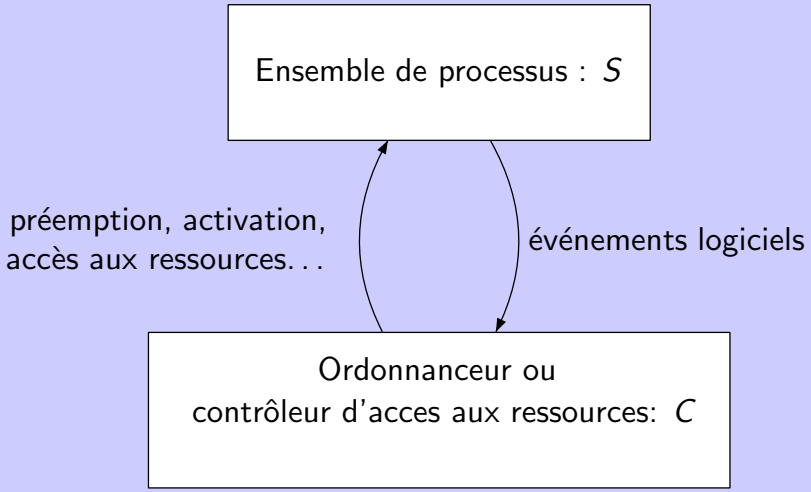

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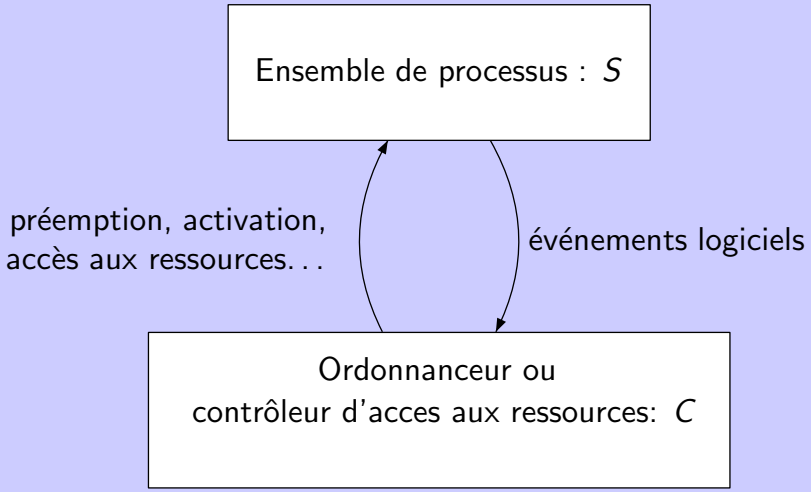

Spécifications :

- Spécification fonctionnelle
- Spécification non fonctionnelle
  - **ressources** (Energie, processeur, mémoire ...)
  - **temporelle**

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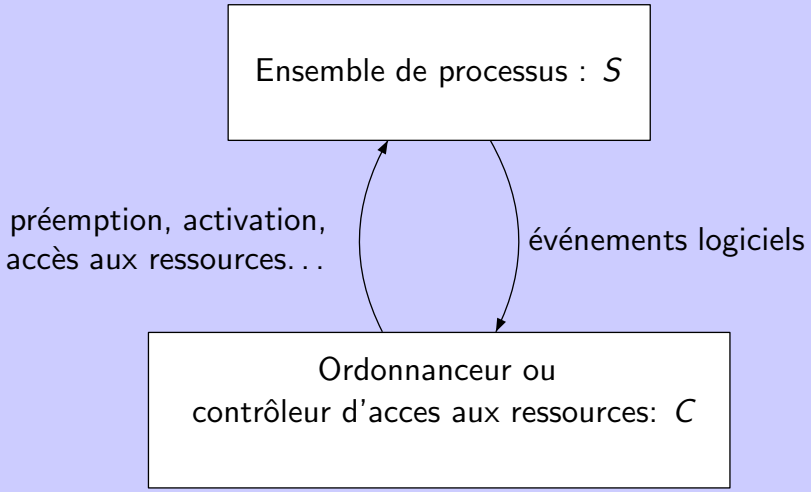

Spécifications :

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- Spécification non fonctionnelle
  - **ressources** (Energie, processeur, mémoire ...)
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Objectif :

Les spécifications doivent être respectées

En entrée :

- Modélisation du système S
- Spécification des propriétés de contrôle : φ

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Problème de la vérification de modèle (model-checking) :

- Modélisation du système contrôlé $S \parallel C$.

$$\text{Est-ce que } S \parallel C \models \varphi \quad ? \tag{1}$$

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Problème du contrôle :

$$\text{Existe-t-il } C \text{ tel que } S \parallel C \models \varphi \quad ? \tag{2}$$

- Si oui : synthétiser ce contrôleur.

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Problème :

Si la réponse est négative, que fait-on ?

Problème de la vérification paramétrée :

- Modélisation paramétrée du système contrôlé $S(p) \parallel C(p)$.

Existe-t-il des valeurs $\nu(p)$ des paramétres p

$$\text{tels que } S(\nu(p)) \parallel C(\nu(p)) \models \varphi(\nu(p)) \quad ? \tag{3}$$

- Si oui : synthétiser l'ensemble des valeurs $\nu(p)$.

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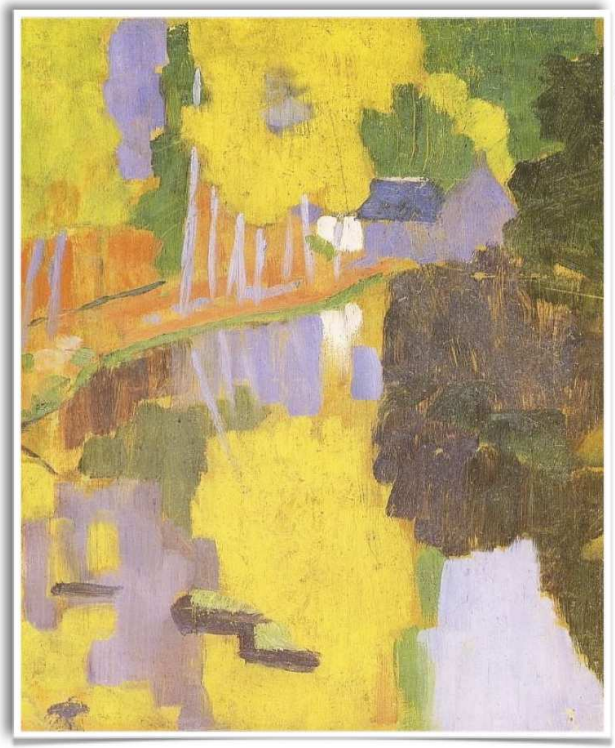
Problème du contrôle paramétré :

Existe-t-il des valeurs $\nu(p)$ des paramétres p et un contrôleur $C((\nu(p))$

$$\text{tels que} S(\nu(p)) \parallel C(\nu(p)) \models \varphi(\nu(p)) \quad ? \tag{4}$$

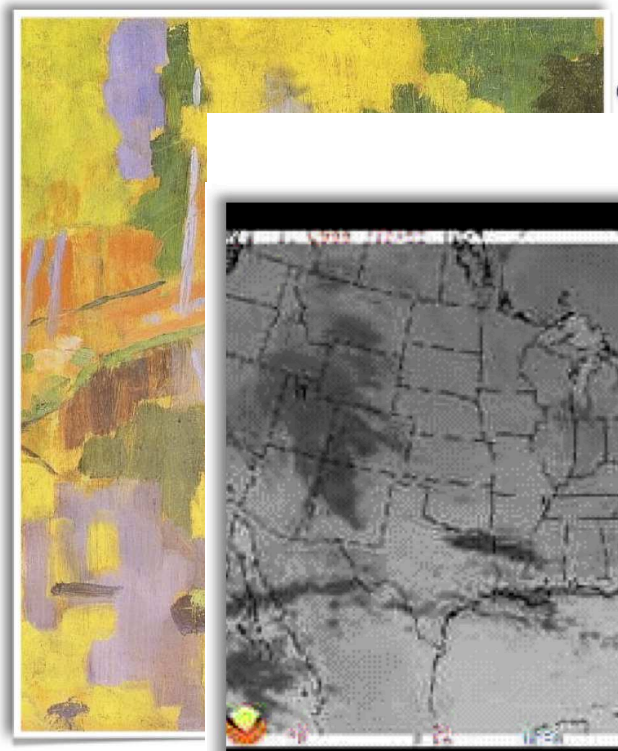
- Si oui : synthétiser l'ensemble des valeurs $\nu(p)$ et le contrôleur $C(\nu(p))$.

Modéliser c'est abstraire



Le Talisman
de P. Sérusier
(1888)

Modéliser c'est abstraire



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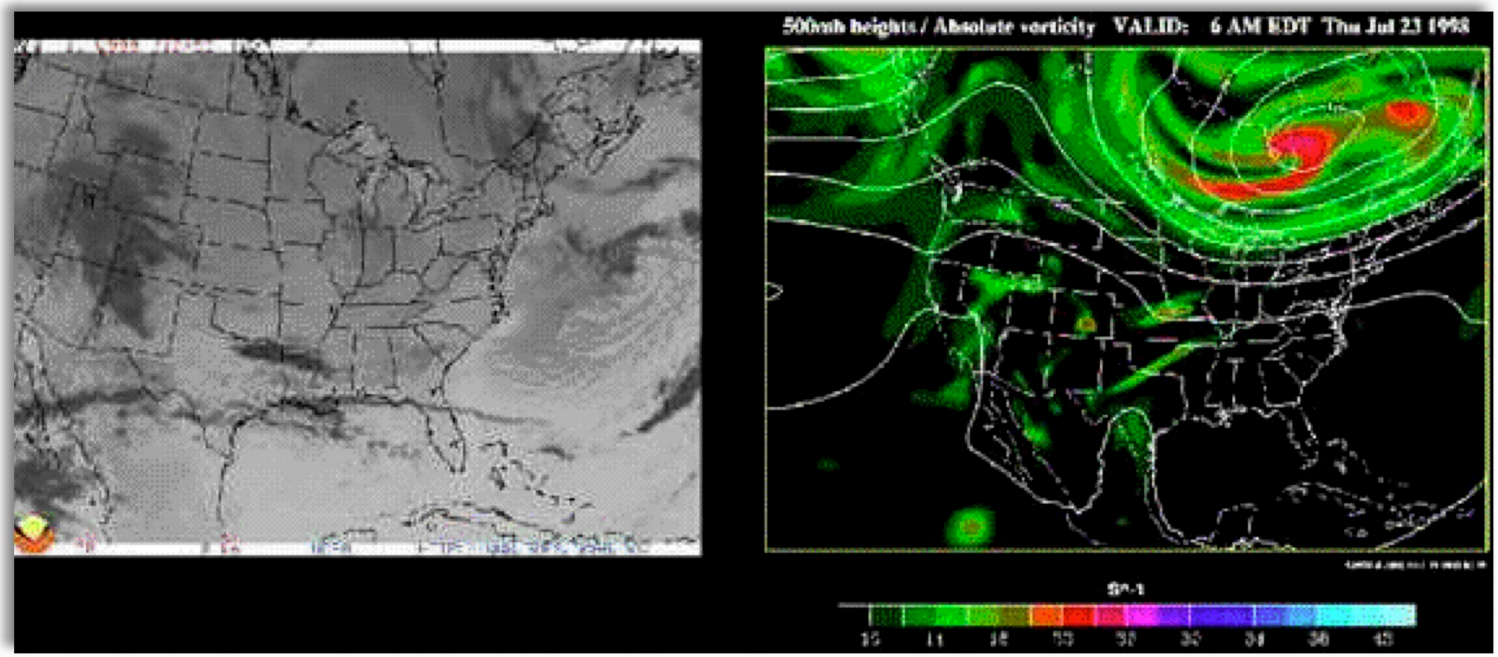
Carte météo



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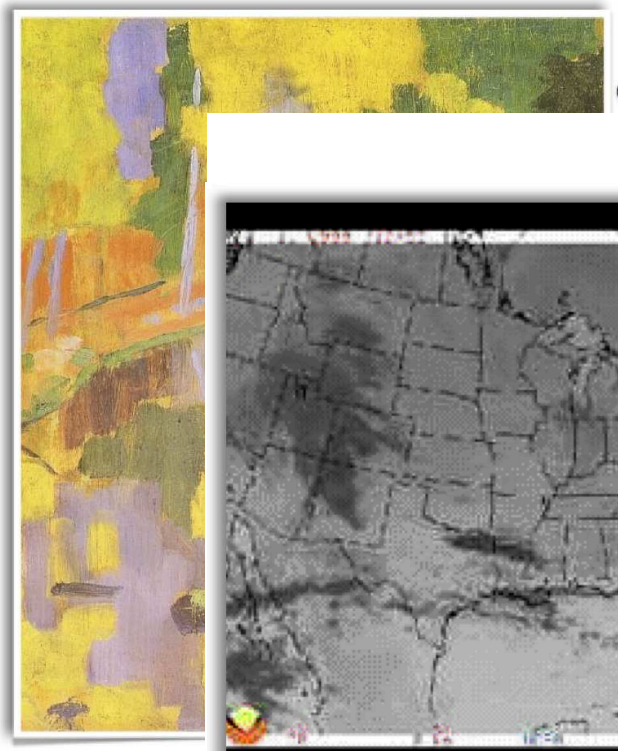

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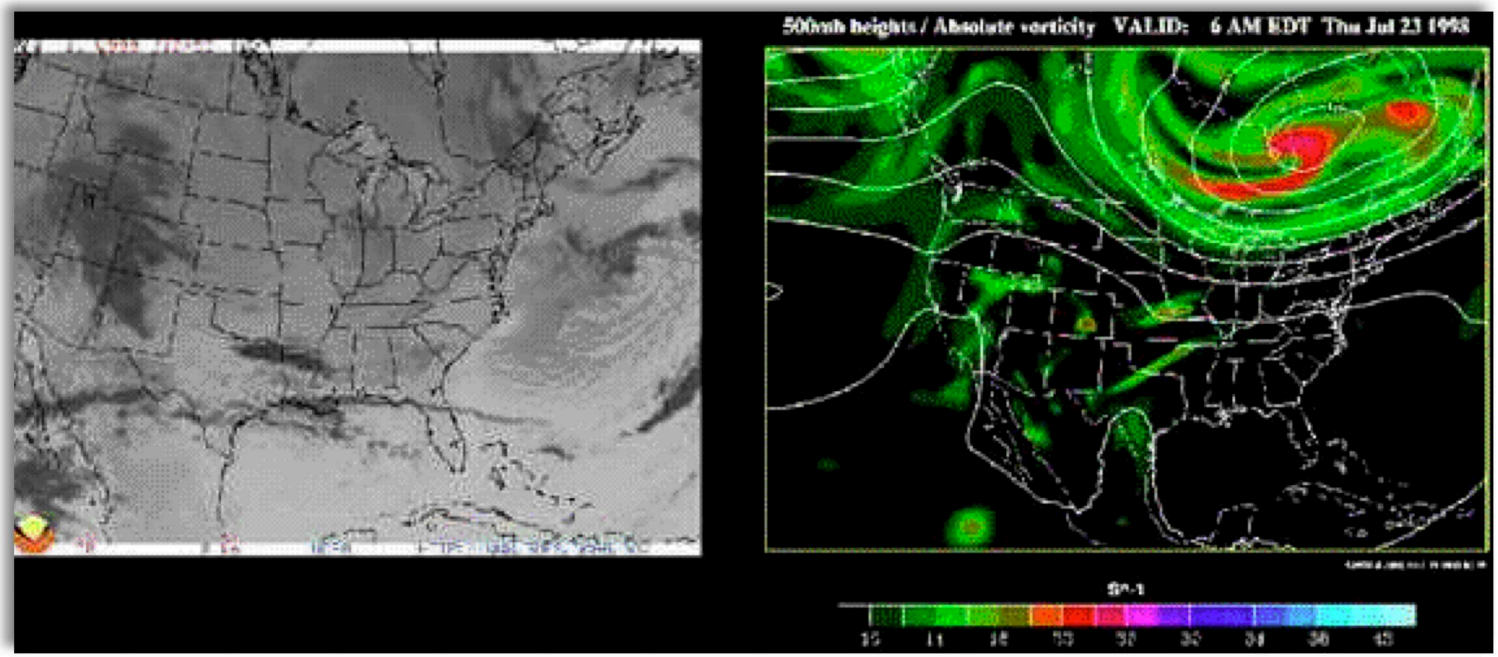

Compromis **Expressivité du modèle - Ce que l'on peut (veut) faire avec**

Applications **temps réel** :

- Spécification **fonctionnelle**
- Spécification **temporelle**

$\Rightarrow$ temps logique généralement insuffisant.

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Extensions **temporisées** des

- Algèbre de processus
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Formalisation de la spécification

- observateurs
- logique temporelle (LTL, CTL)
- logique temporelle temporisée (TCTL)

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Contraintes temporelles paramétrées sur

le modèle et/ou la spécification

Outline

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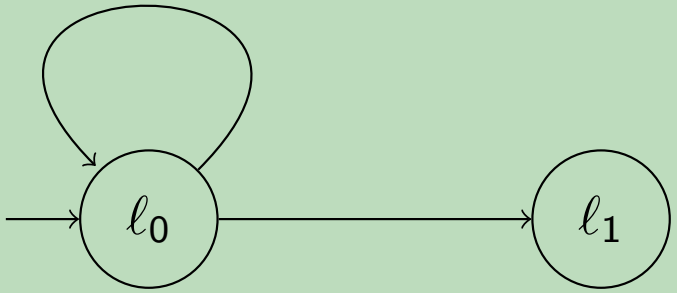
Timed Automata

A timed automaton is made of

- a transition system

Example



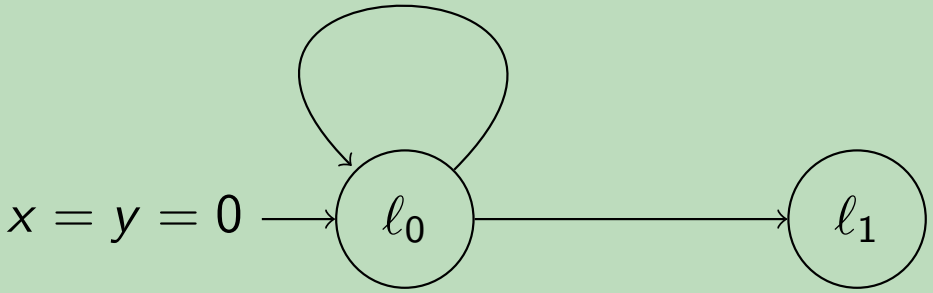
Timed Automata

A timed automaton is made of

- a transition system
- a set of clocks

Example

Clocks (x, y):



Timed Automata

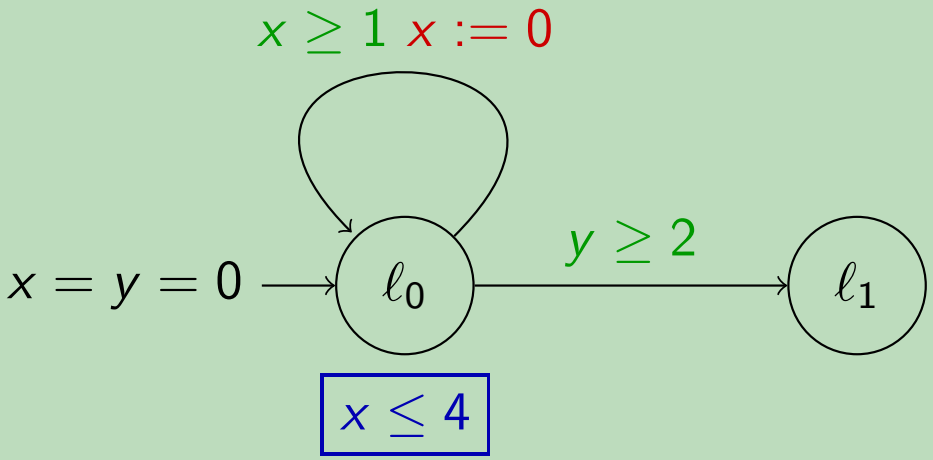
A timed automaton is made of

- a transition system
- a set of clocks
- timing constraints on locations and edges

Example

Clocks (x, y):

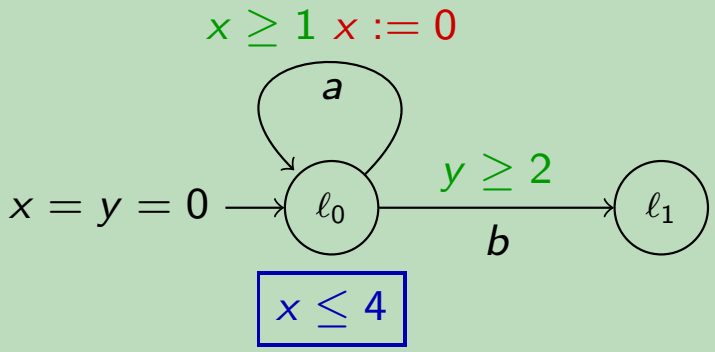
- guards
- invariants
- resets

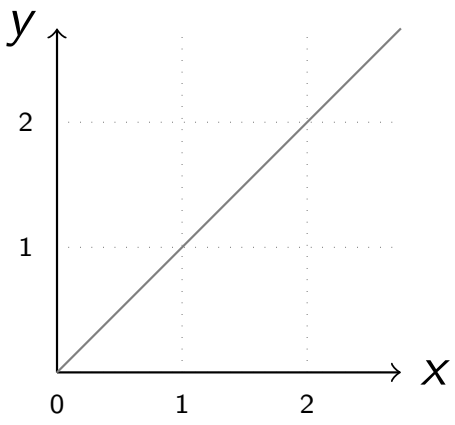


Timed Automata (TA) [AD94]

Exemple

- a transition system
- a set of clocks (here: x, y)
- timing constraints





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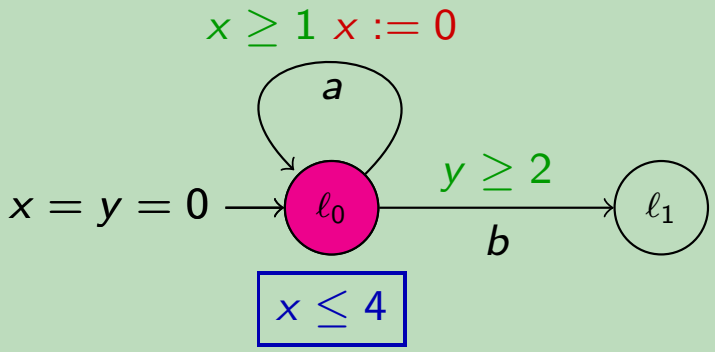
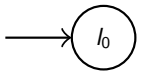
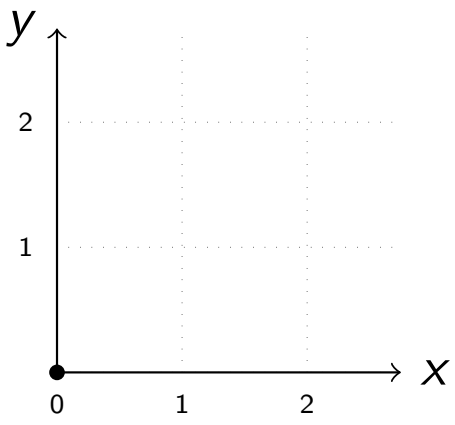




$(0, 0)$



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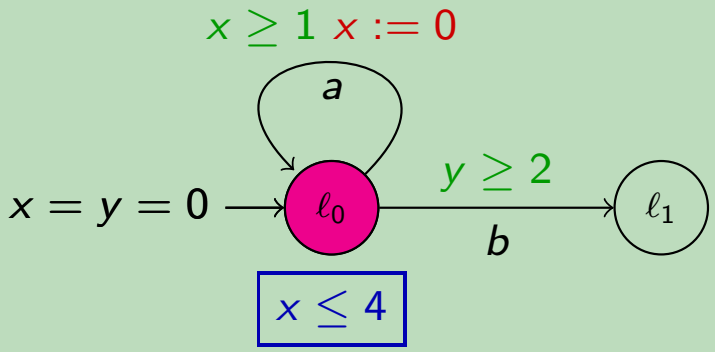
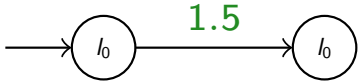
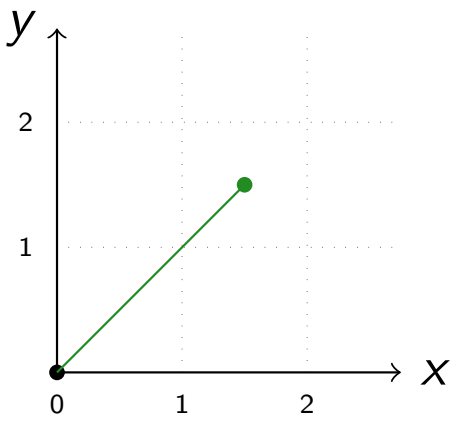




$(0, 0)$ $(1.5, 1.5)$



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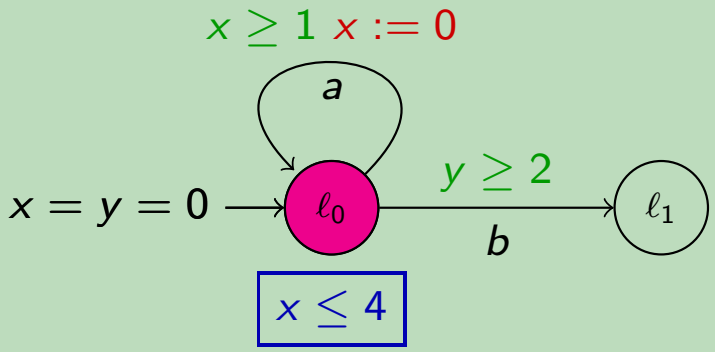

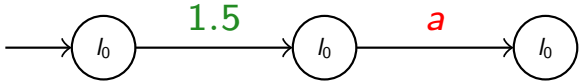

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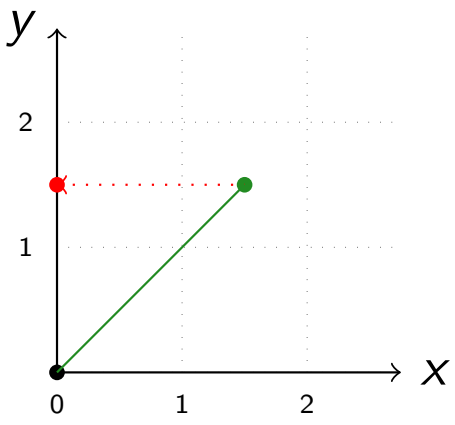

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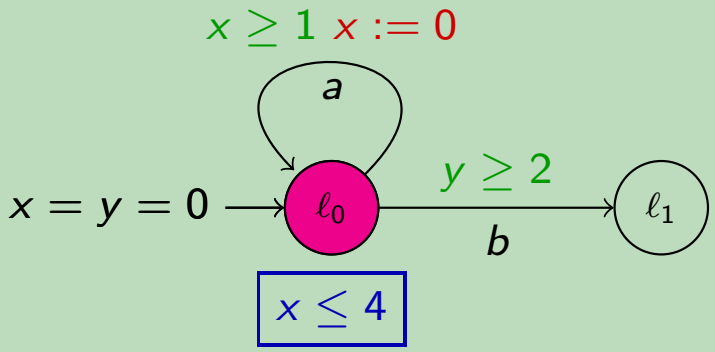

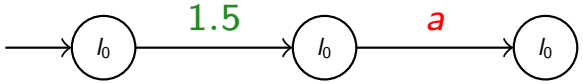

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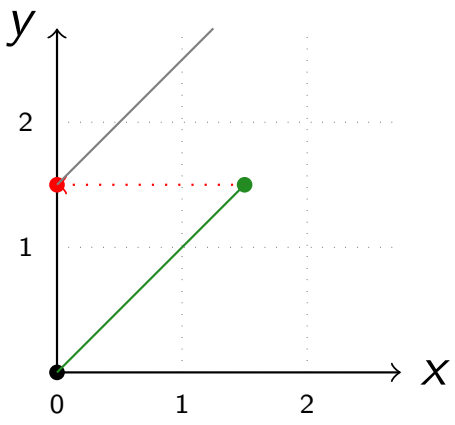

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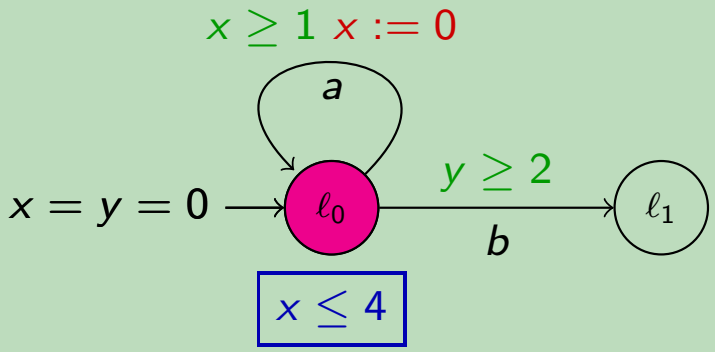

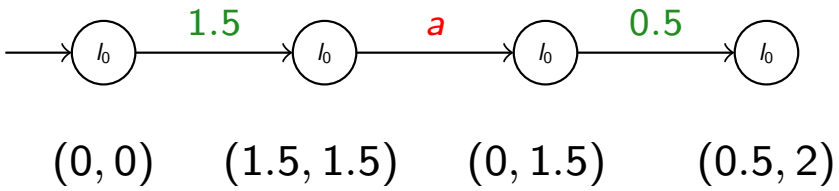

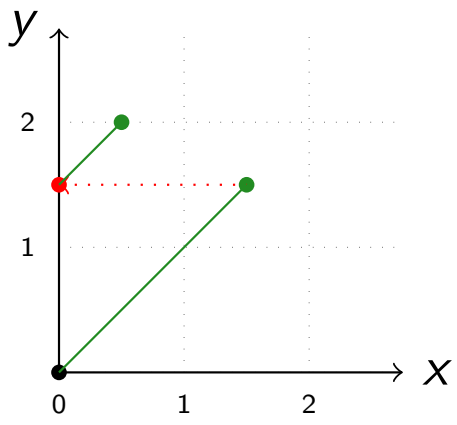

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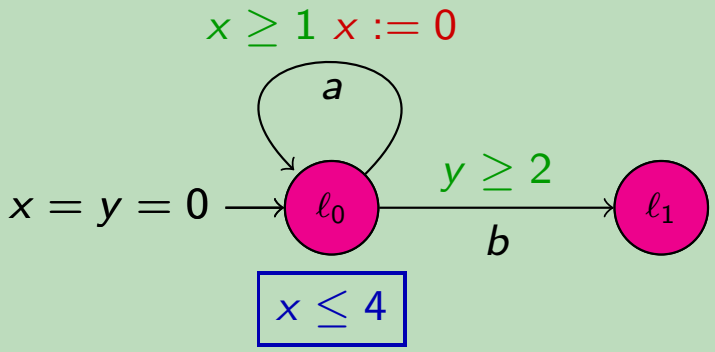

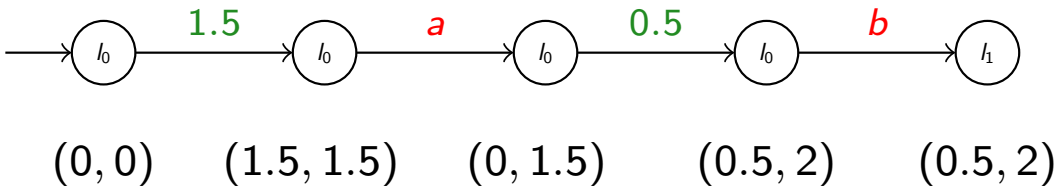

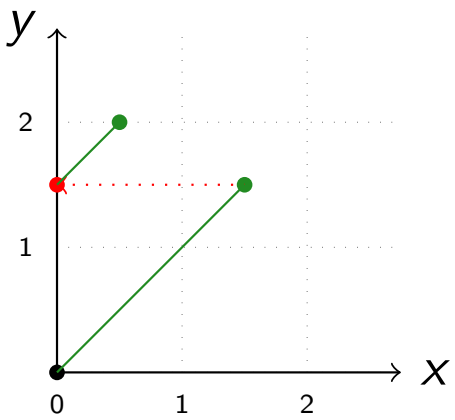

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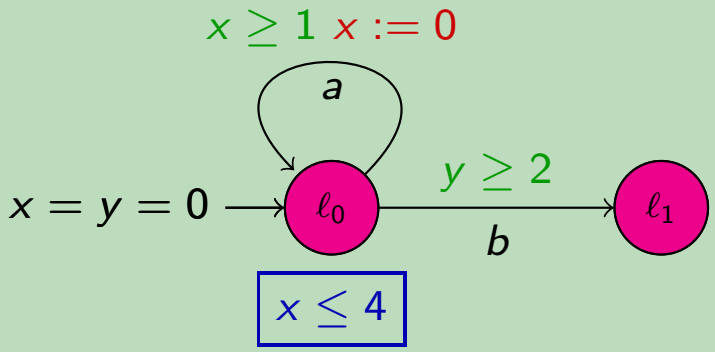

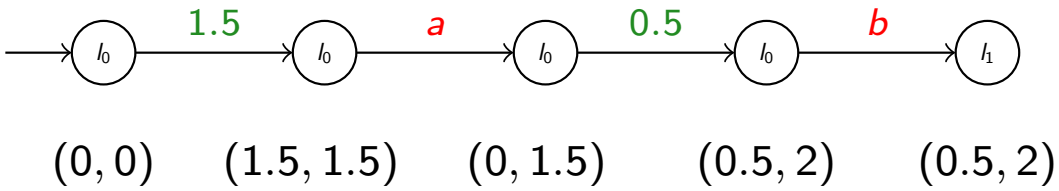

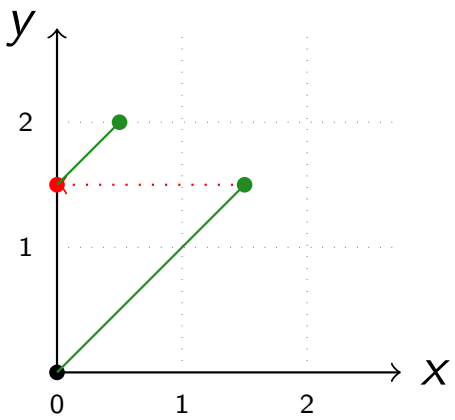

Infinité de branchements et d'états

Abstractions symboliques de l'espace d'états : graphe des **régions**, graphe des **zones**.

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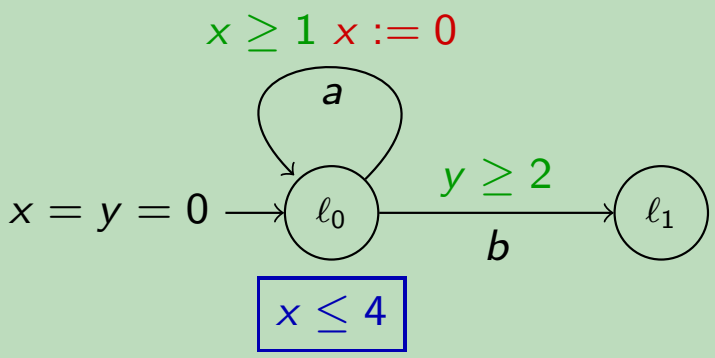
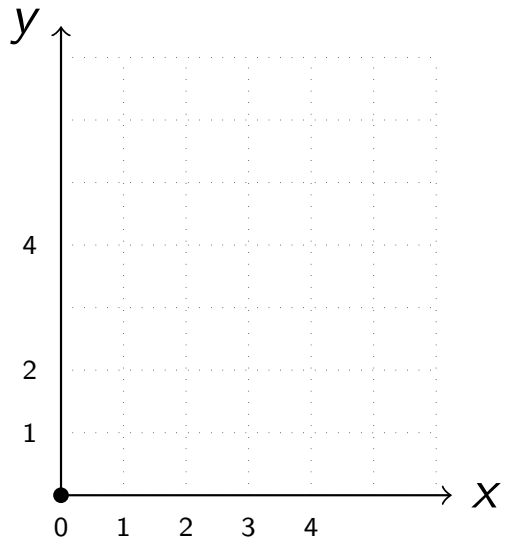


ℓ_0



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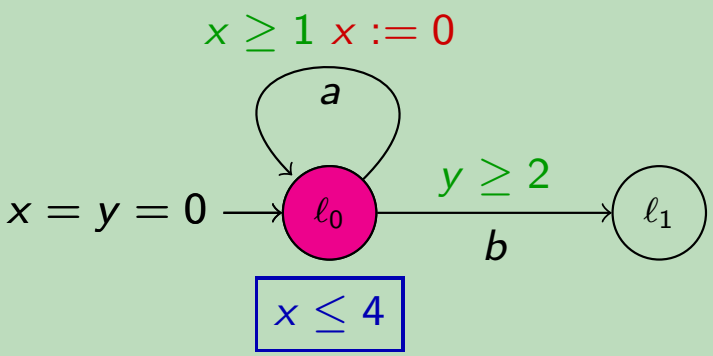
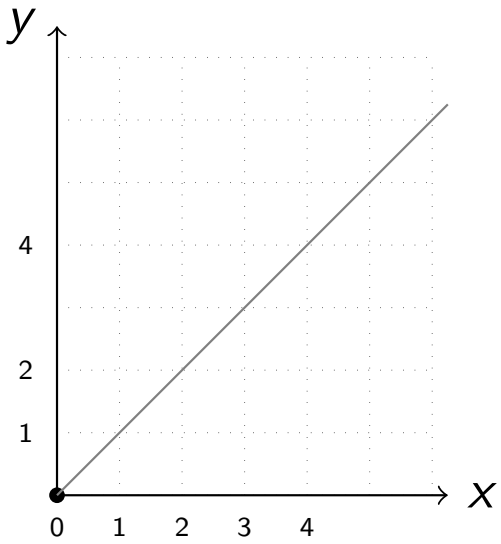


ℓ_0
$x = y$



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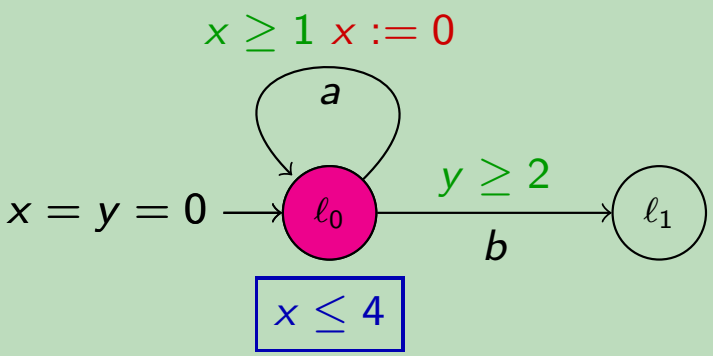
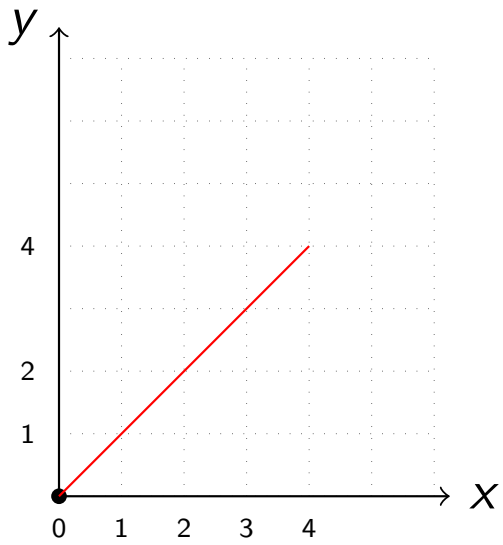


ℓ_0
$x = y$
$x \leq 4$



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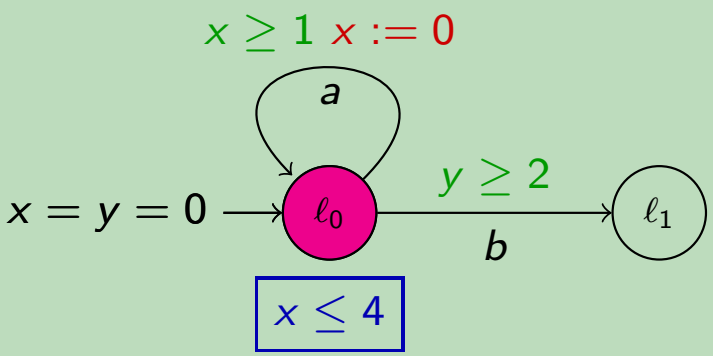
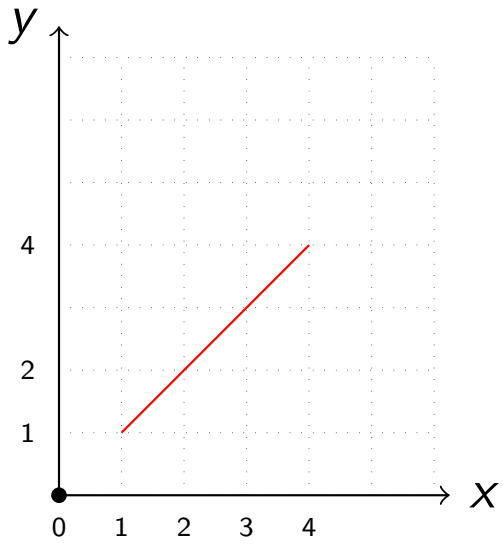


ℓ_0
$x = y \quad \xrightarrow{a}$
$x \leq 4$



Timed Automata (TA) [AD94]

Exemple

- a transition system
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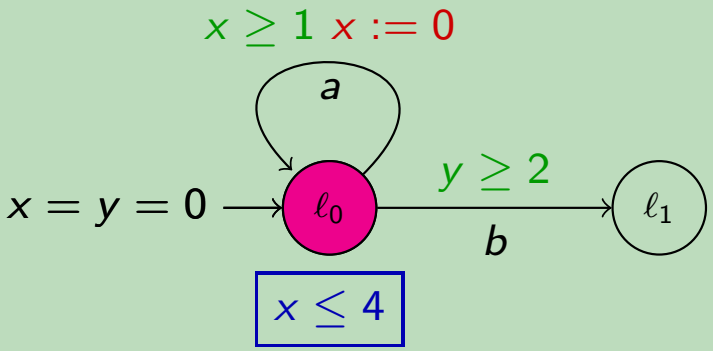
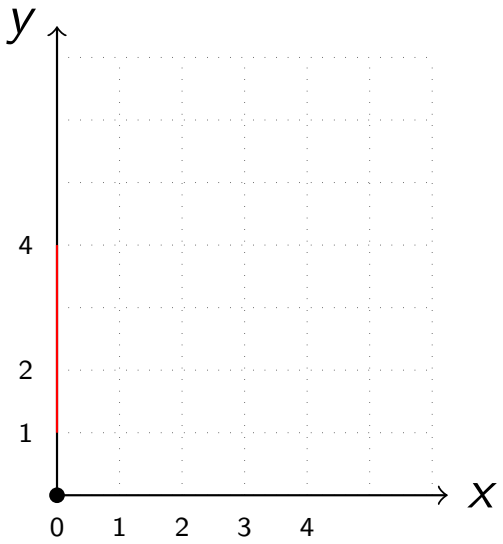


$$\begin{array}{lcl} \ell_0 & & \ell_0 \\ x = y & \xrightarrow{a} & 1 \leq y - x \leq 4 \\ x \leq 4 & & x = 0 \end{array}$$



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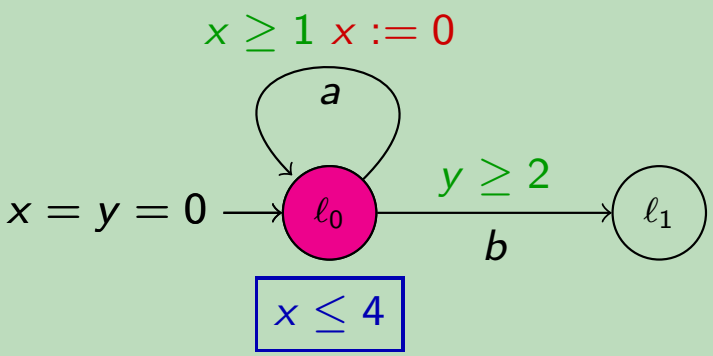

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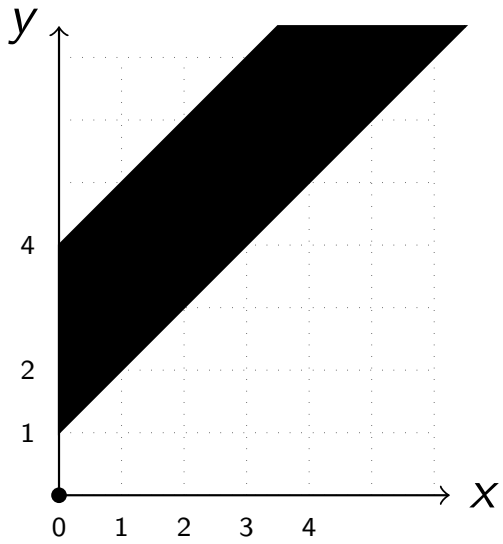

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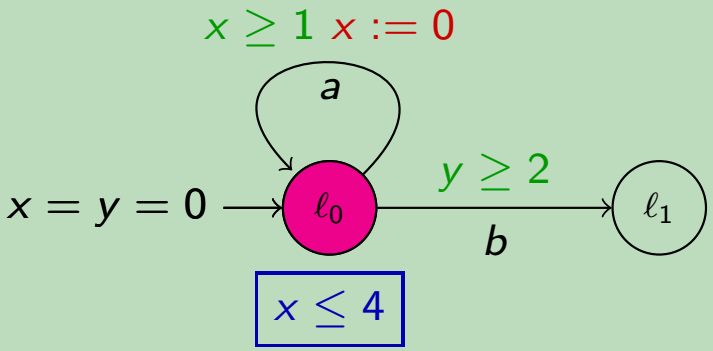

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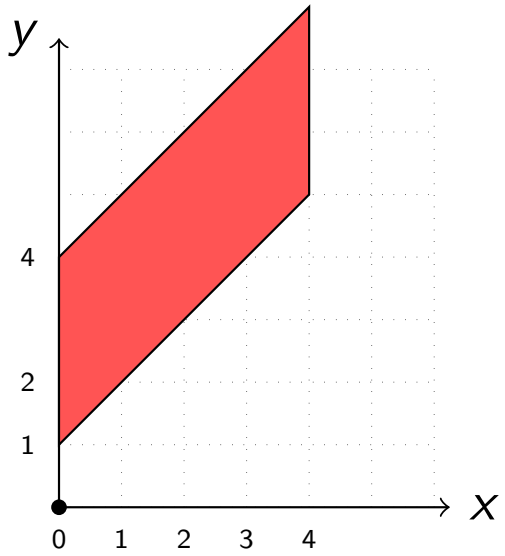

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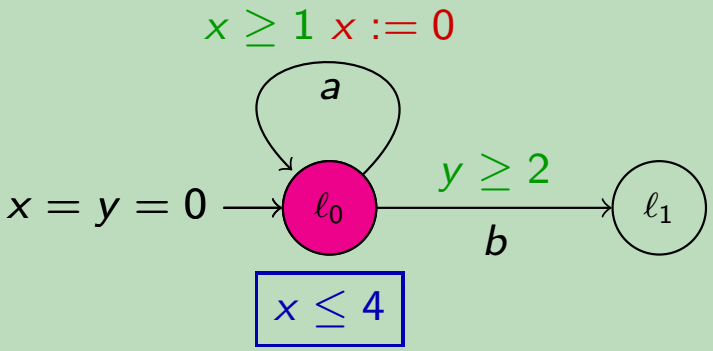
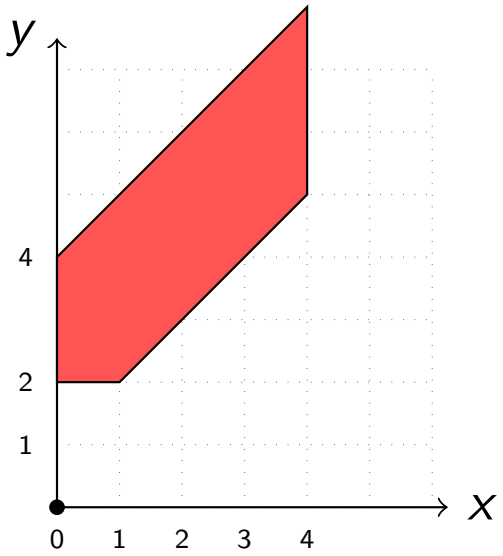


$$\begin{array}{l} \ell_0 \\ x = y \\ x \leq 4 \end{array} \xrightarrow{a} \begin{array}{l} \ell_0 \\ 1 \leq y - x \leq 4 \\ x \leq 4 \end{array} \xrightarrow{b}$$



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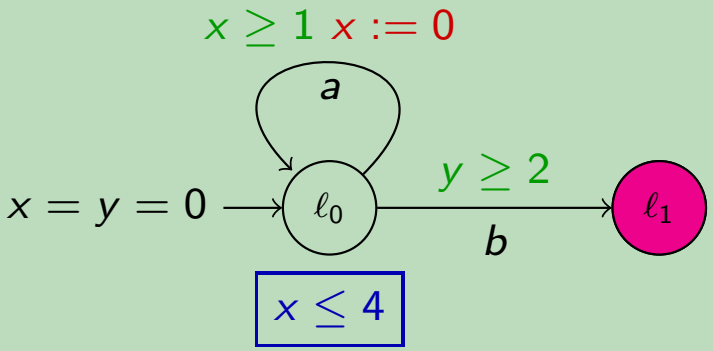
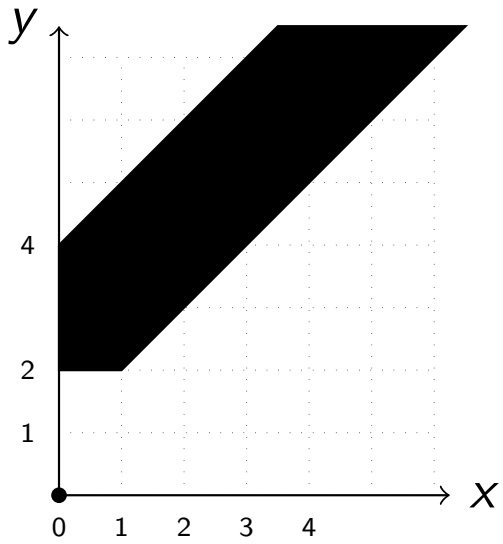


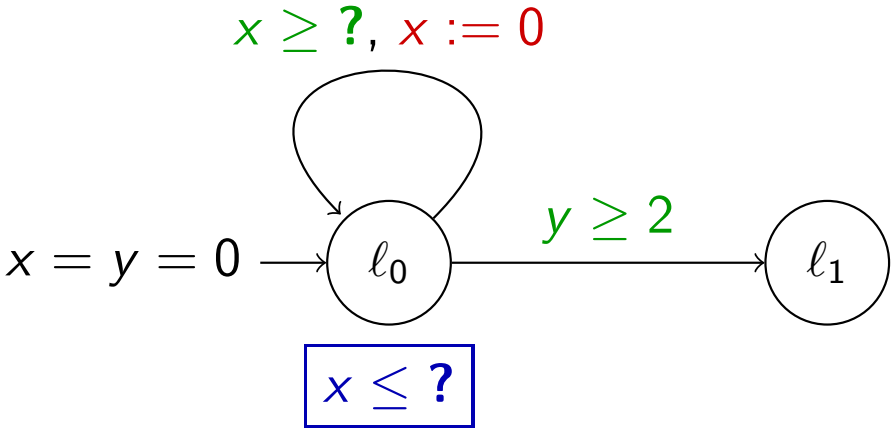
$$\begin{array}{l} \ell_0 \\ x = y \\ x \leq 4 \end{array} \xrightarrow{a} \begin{array}{l} \ell_0 \\ 1 \leq y - x \leq 4 \\ x \leq 4 \end{array} \xrightarrow{b} \begin{array}{l} \ell_1 \\ 1 \leq y - x \leq 4 \\ y \geq 2 \end{array}$$



Theorem [AD94]

L'accessibilité est décidable pour les TA et PSPACE-complet.

Parametric Timed Automata (PTA) [AHV93]



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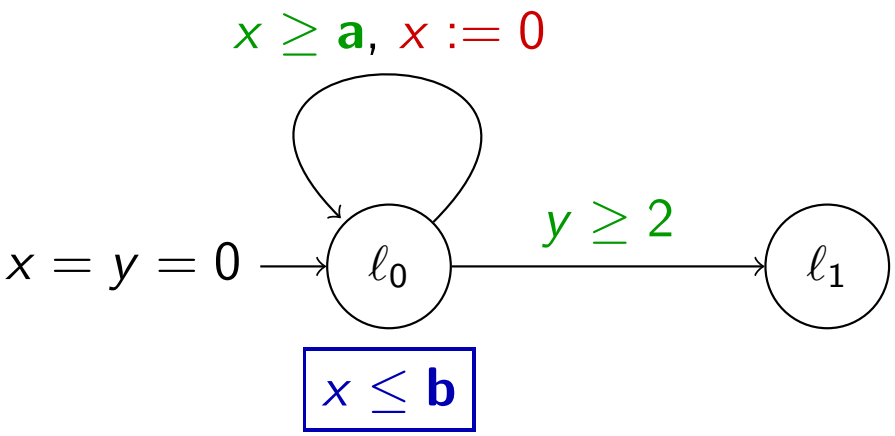

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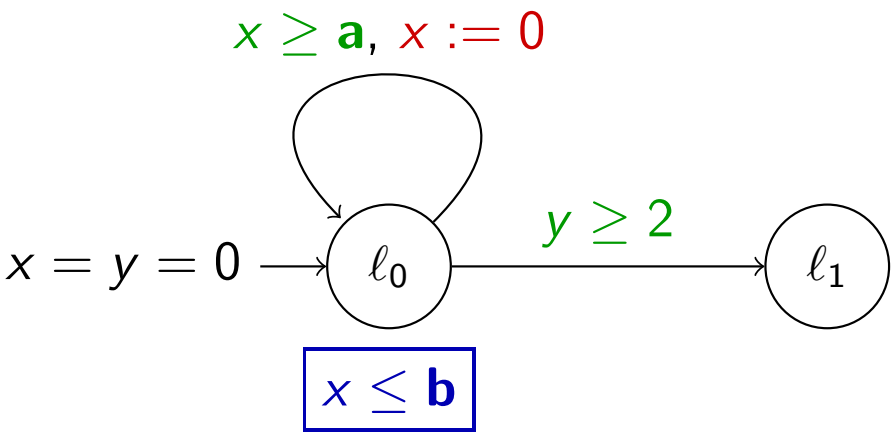

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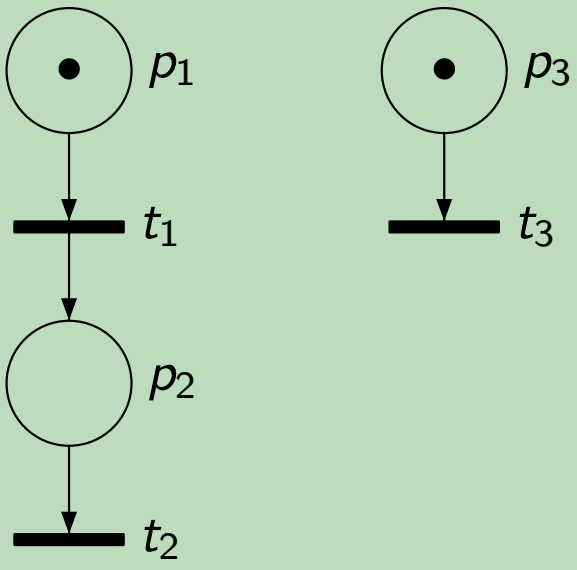
- Pour un PTA $\mathcal{A}$ et une valuation des parametres $v : P \to \mathbb{Q}$, "$v(\mathcal{A})$" est un a **automate temporisé**.

Réseau de Petri T-temporel (TPN)

Exemple



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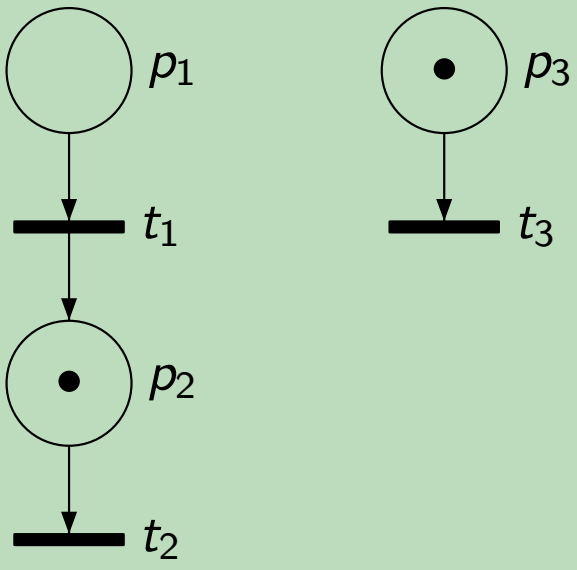

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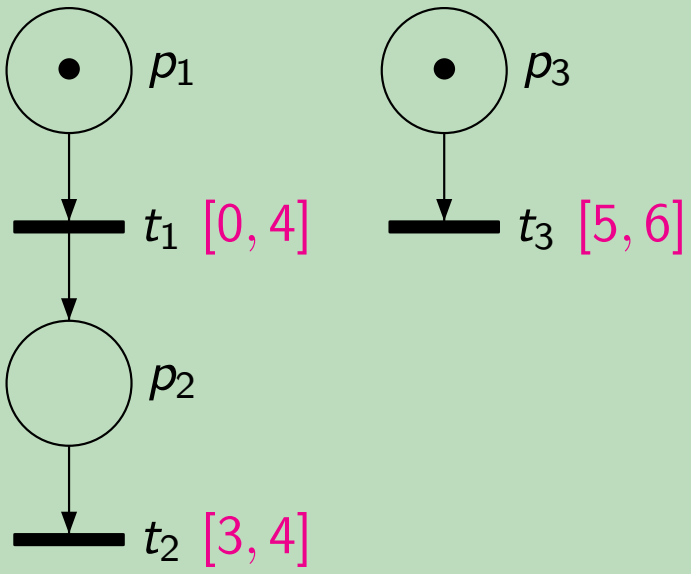

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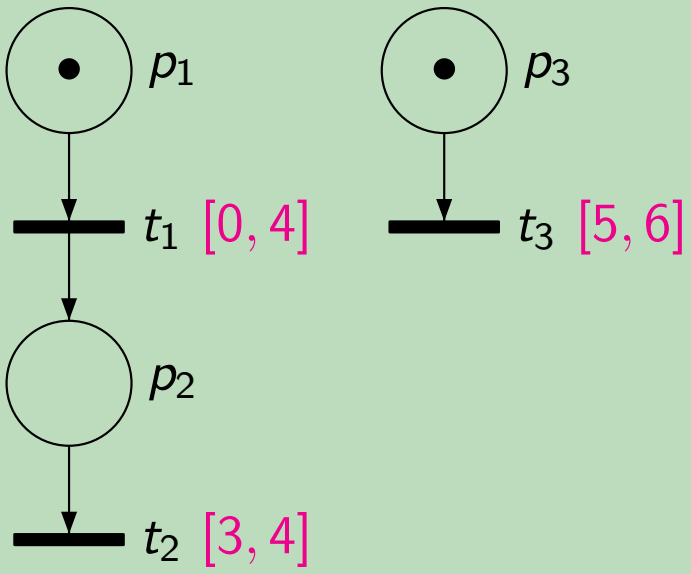
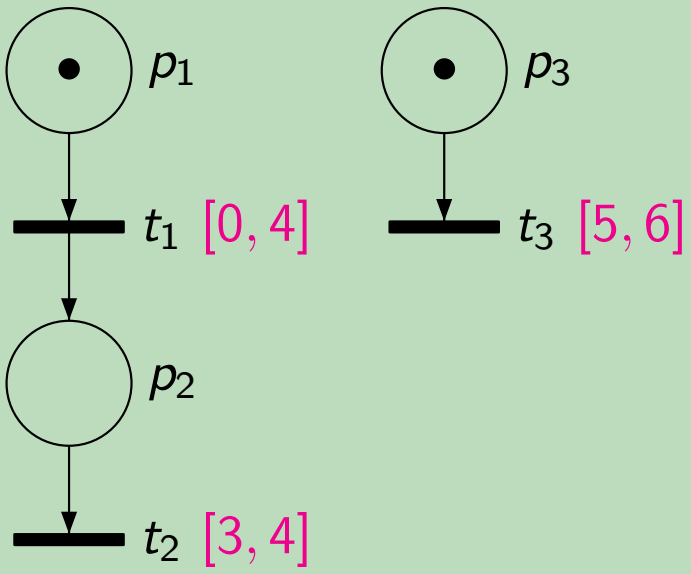


$\{p_1, p_3\}$
$\nu(t_1) = 0$
$\nu(t_3) = 0$

Réseau de Petri T-temporel (TPN)

Exemple



$$\begin{array}{lcl} \{p_1, p_3\} & & \{p_1, p_3\} \\ \nu(t_1) = 0 & \xrightarrow{4} & \nu(t_1) = 4 \\ \nu(t_3) = 0 & & \nu(t_3) = 4 \end{array}$$

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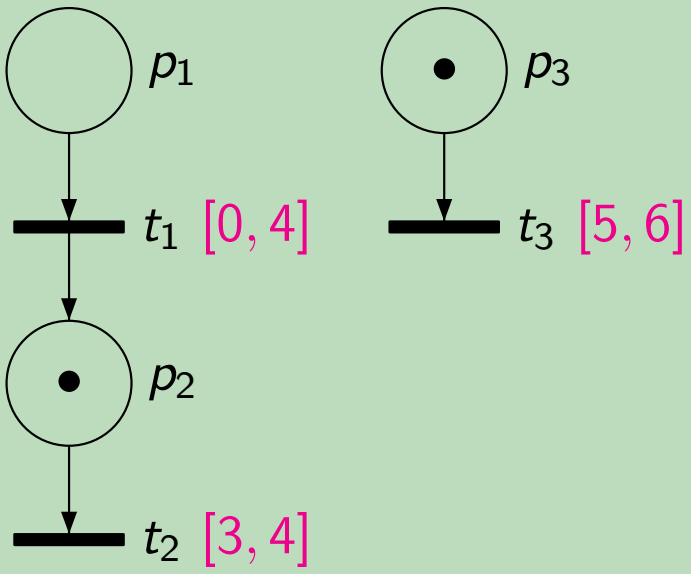
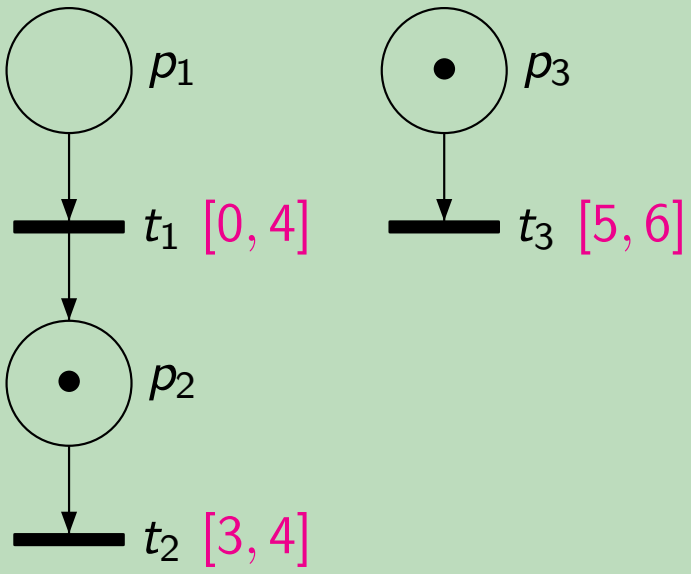
Exemple



$$\begin{array}{c} \{p_1, p_3\} \\ \nu(t_1) = 0 \\ \nu(t_3) = 0 \end{array} \xrightarrow{4} \begin{array}{c} \{p_1, p_3\} \\ \nu(t_1) = 4 \\ \nu(t_3) = 4 \end{array} \xrightarrow{t_1} \begin{array}{c} \{p_2, p_3\} \\ \nu(t_2) = 0 \\ \nu(t_3) = 4 \end{array}$$

Réseau de Petri T-temporel (TPN)

Exemple



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Réseau de Petri T-temporel (TPN)

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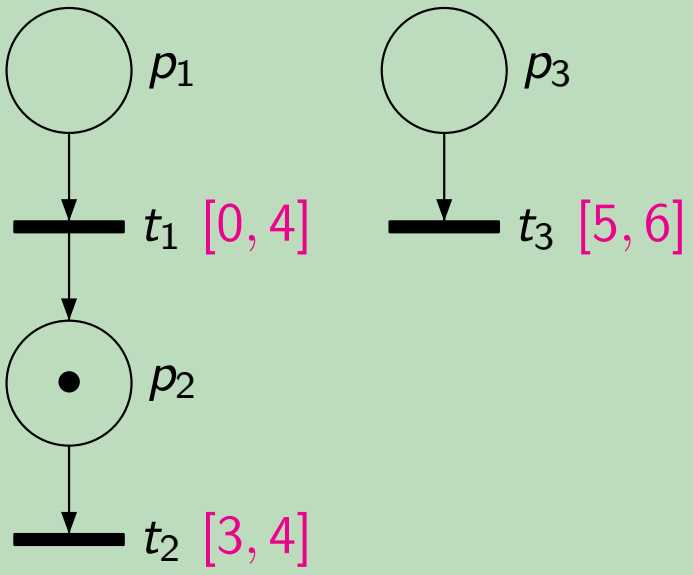

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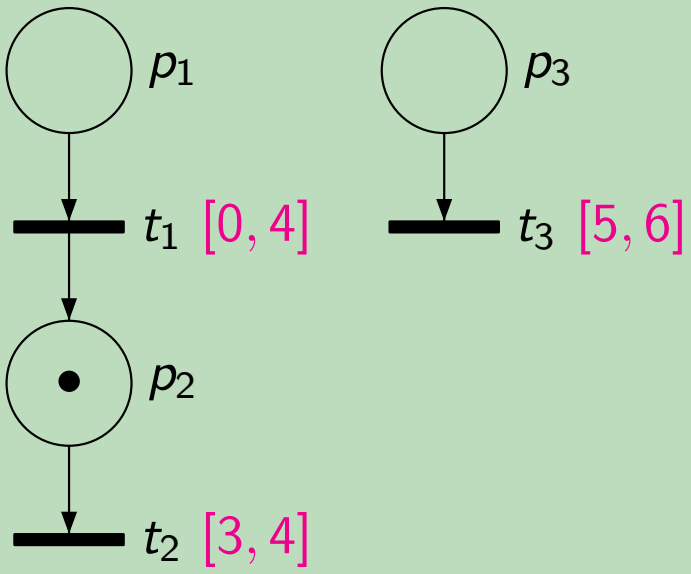
Abstractions symboliques de l'espace d'états

Graphe des **classes**, graphe des **régions**, graphe des **zones**

Réseau de Petri T-temporel (TPN)

Exemple



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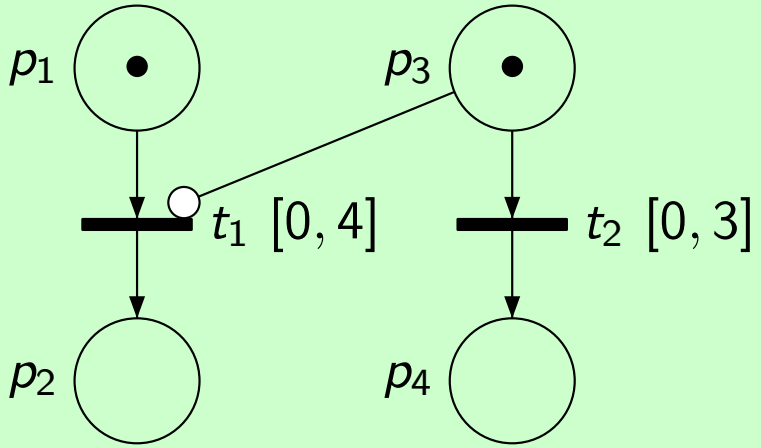
Theorem [BD91]

L'accessibilité est décidable pour les TPN bornés et PSPACE-complet.

Des arcs particuliers

Example (Arc inhibiteur logique)



Peut-on tirer t_1 avant t_2 ?

Des arcs particuliers

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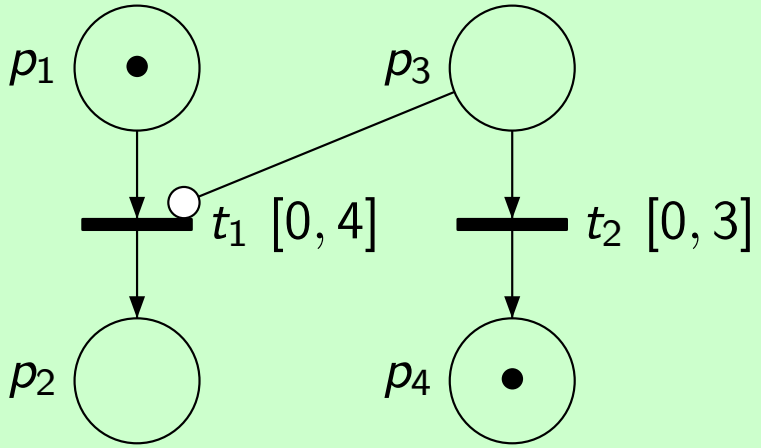

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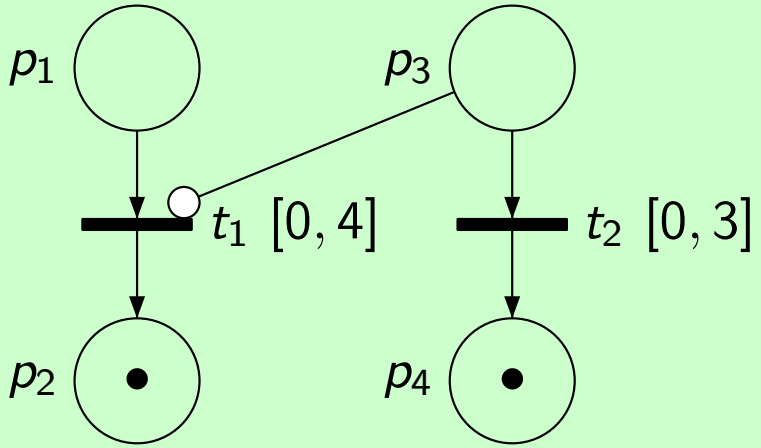

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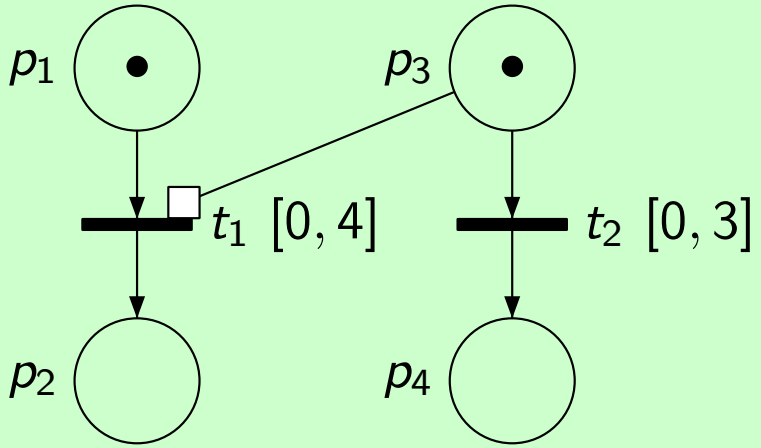
Des arcs particuliers

Example (Arc de lecture)



Peut-on tirer t_2 avant t_1 ?

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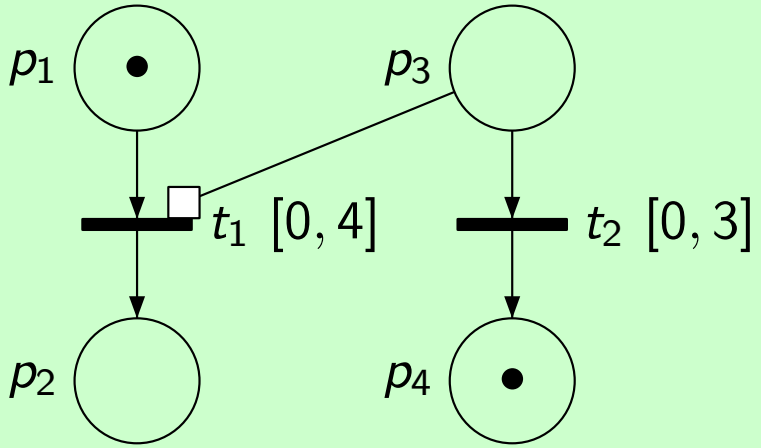

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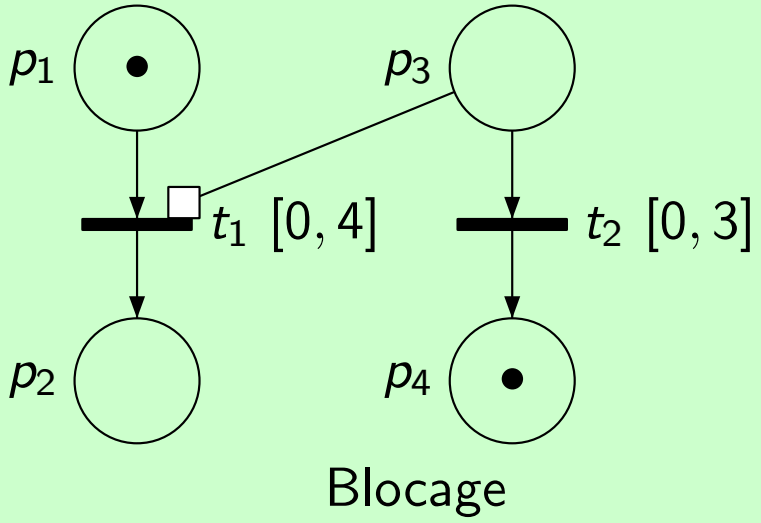

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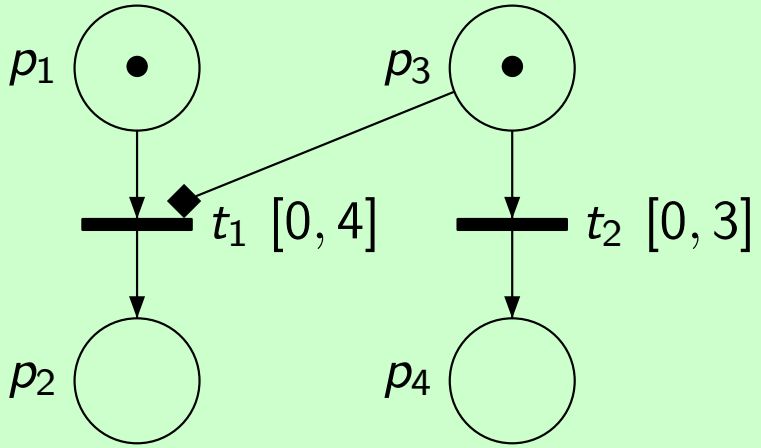
Example (Arc de vidange)



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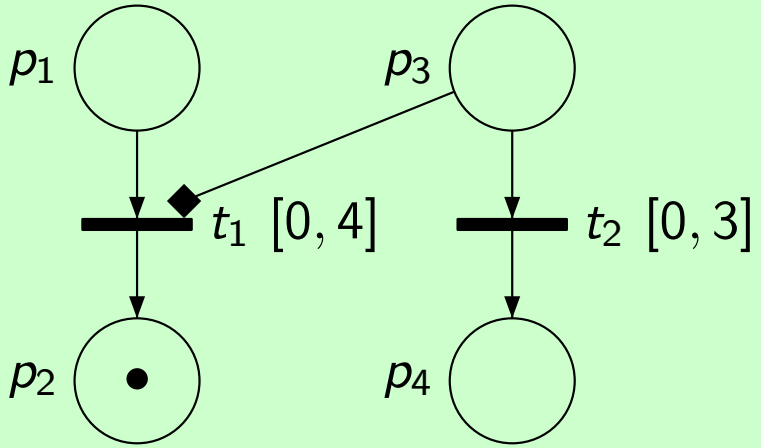

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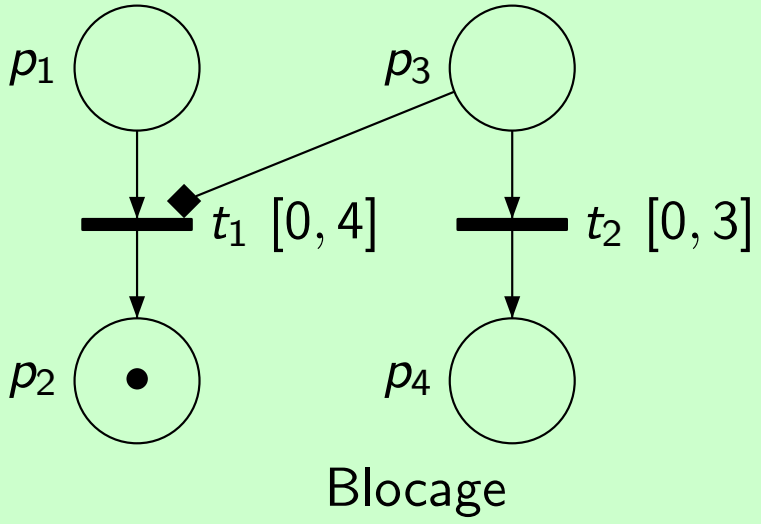
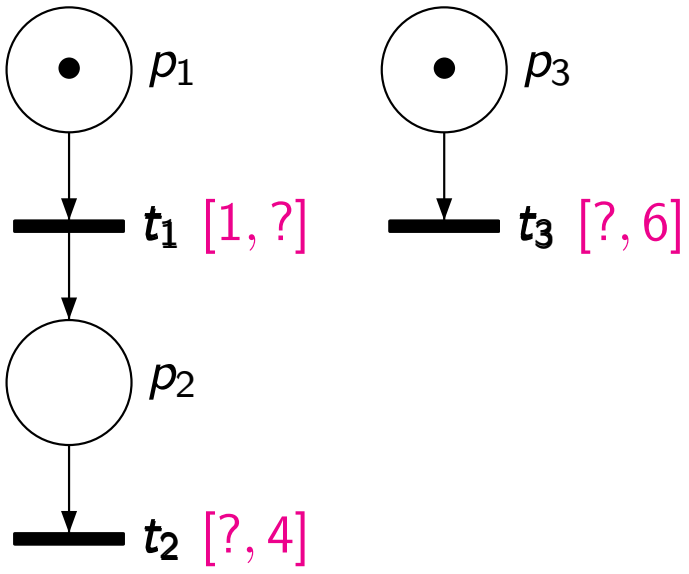


Peut-on tirer t_1 avant t_2 ?

Parametric Time Petri Nets (PTPN) [TLR09]



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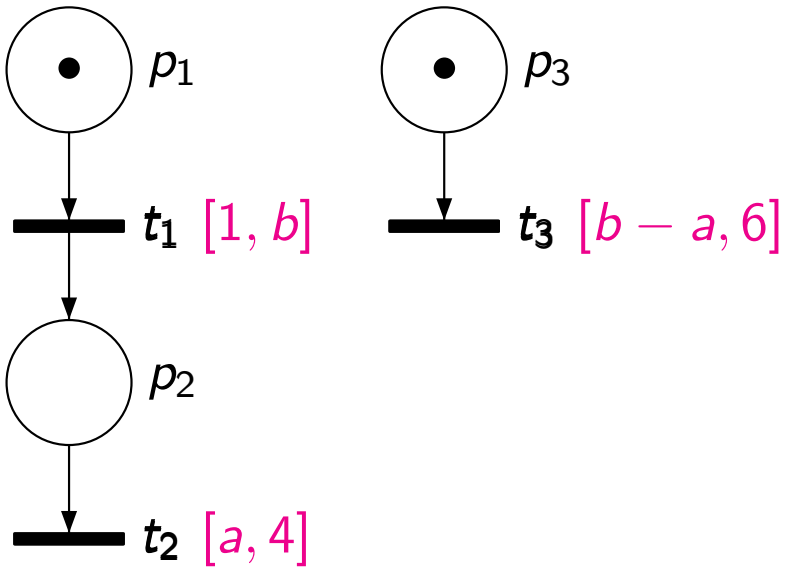

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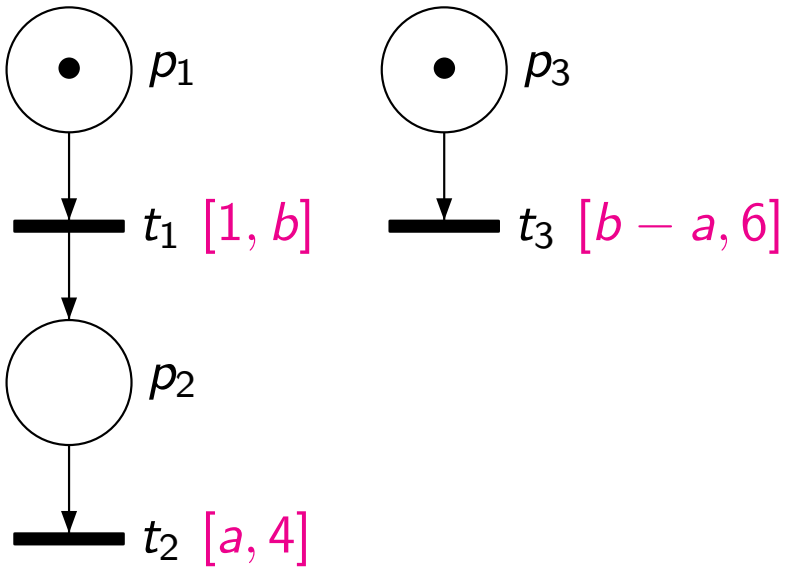

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- Pour un PTPN $\mathcal{N}$ et une valuation des parametres $v : P \to \mathbb{Q}$, "$v(\mathcal{N})$" est un **réseau de Petri temporel**.

Outline

Les problèmes paramétrés

- Soit φ une propriété et $\mathcal{A}$ un PTA (ou un PTPN)
- Soit $V_\varphi(\mathcal{A})$ l'ensemble des valeurs v des paramètres t.q.. $v(\mathcal{A}) \models \varphi$.

φ-emptiness problem

L'ensemble $V_\varphi(\mathcal{A})$ est-il vide ?

φ-synthesis problem

Pouvons nous synthétiser l'ensemble des valeurs des paramètres $V_\varphi(\mathcal{A})$?

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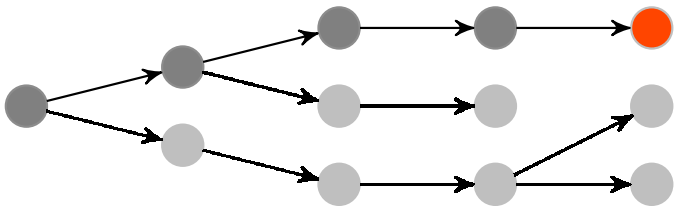

(a) Reachability (EF)

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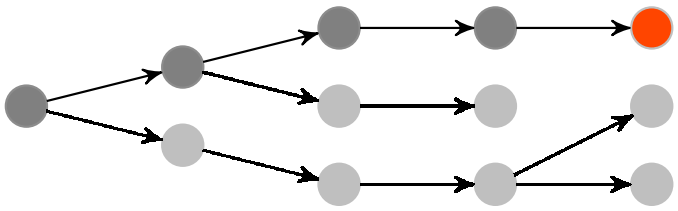
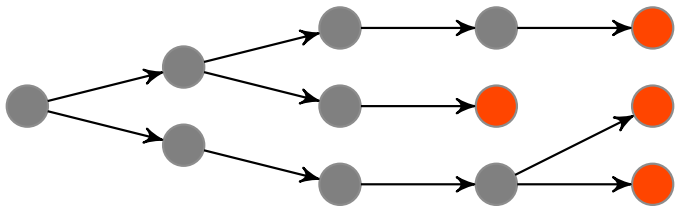


(a) Reachability (EF)



(b) Unavoidability (AF)

Résultats de (In)décidabilité

Theorem [AHV93]

The EF-emptiness problem for PTA is **undecidable**.

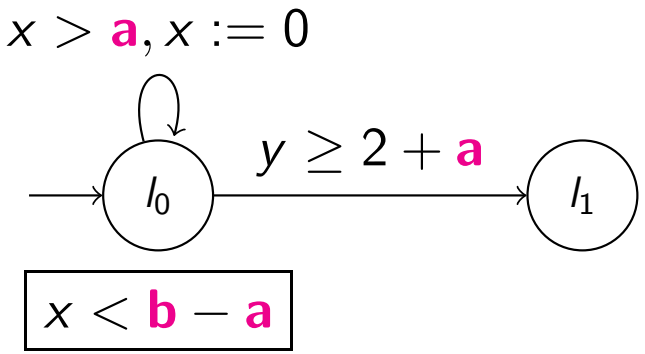
Proof:

- Reduction from the 2-counter machine halting problem
- PTA is constructed such that the machine halts iff $V \neq \emptyset$
- If the machine does not halt, the PTA cannot reach l_{halt}: $V = \emptyset$;

L/U-automata [HRSV02]

- In L/U-automata, parameters are either **upper bounds** (e.g. $x \leq b$ or $x \geq 1 - b$) or **lower bounds** on clocks.

L/U

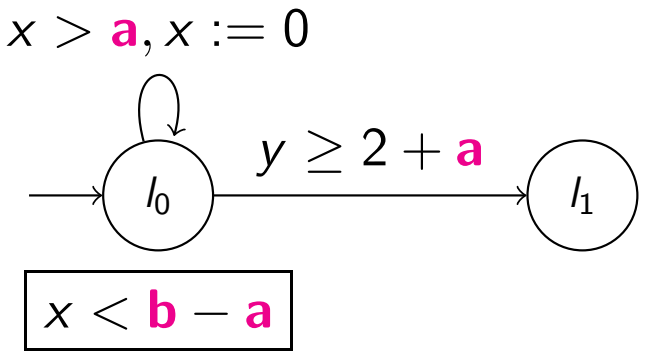


- **All** the possible behaviors can be obtained by setting lower bounds to 0 and upper bounds to $+\infty$.

L/U-automata [HRSV02]

- In L/U-automata, parameters are either **upper bounds** (e.g. $x \leq b$ or $x \geq 1 - b$) or **lower bounds** on clocks.

L/U



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Theorem [HRSV02]

The EF-emptiness problem for L/U-automata is decidable and PSPACE-complete.

L/U-automata: AF-Emptiness

Theorem [JLR13a]

The AF-emptiness problem for **U**-automata is undecidable.

- Proved by adapting the 2-counter machine reduction of [AHV93].
- Machine halts iff l_{halt} is unavoidable in the U-automaton

L/U-automata: Synthesis

Theorem [JLR13a]

For L/U automata, the solution to the EF-synthesis problem cannot be represented using any formalism for which emptiness of the intersection with equality constraints is decidable.

(in particular, not **finite unions of convex polyhedra**)

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Proof (inspired from [BT09]):

- In a PTA, if p_i is both a lower bound and an upper bound:
  - Replace p_i by a fresh parameter p_i^u when p_i is an upper-bound;
  - Replace p_i by a fresh parameter p_i^l when p_i is a lower-bound.
- $V' = \bigcap_i p_i^u = p_i^l$

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Theorem [JLR13a]

EF-emptiness for possibly **unbounded integer** values is undecidable.

Proof: As in [AHV93]

Outline

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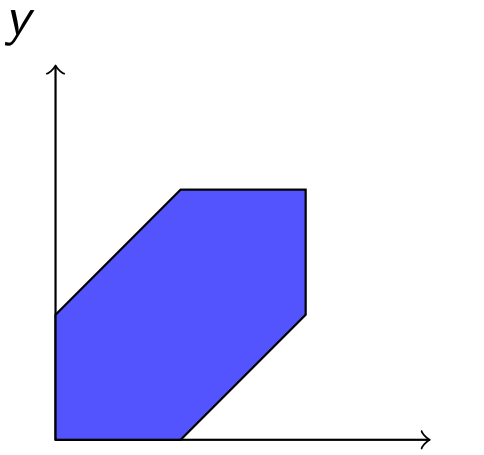

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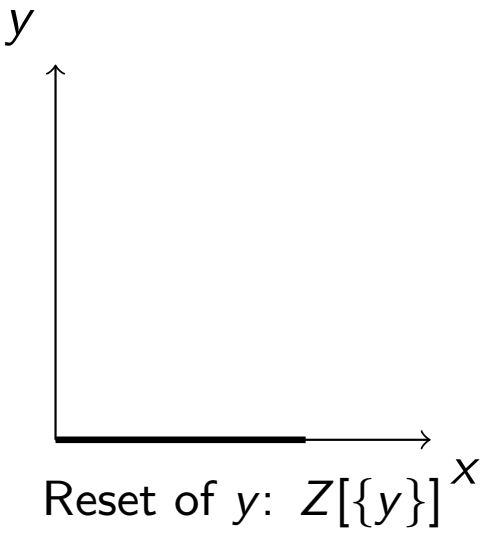

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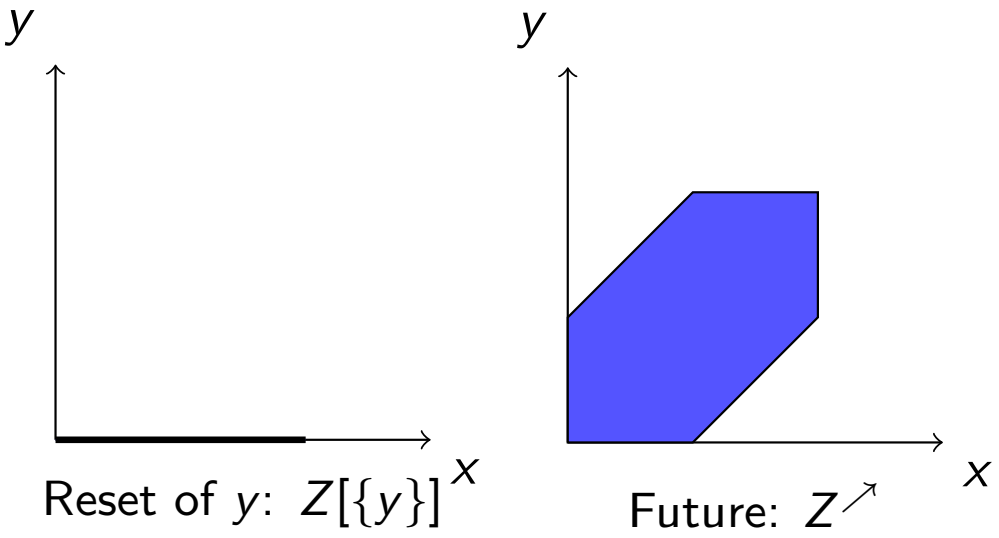

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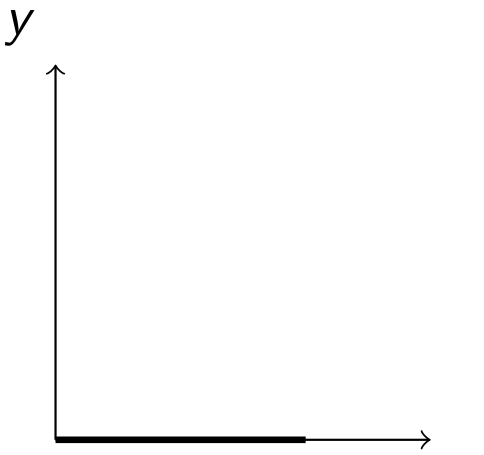

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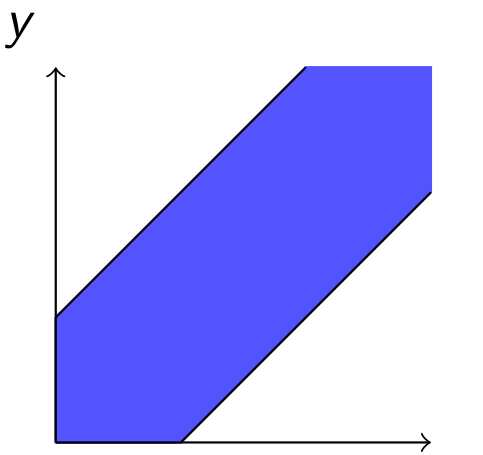

Future: $Z^{\nearrow}$

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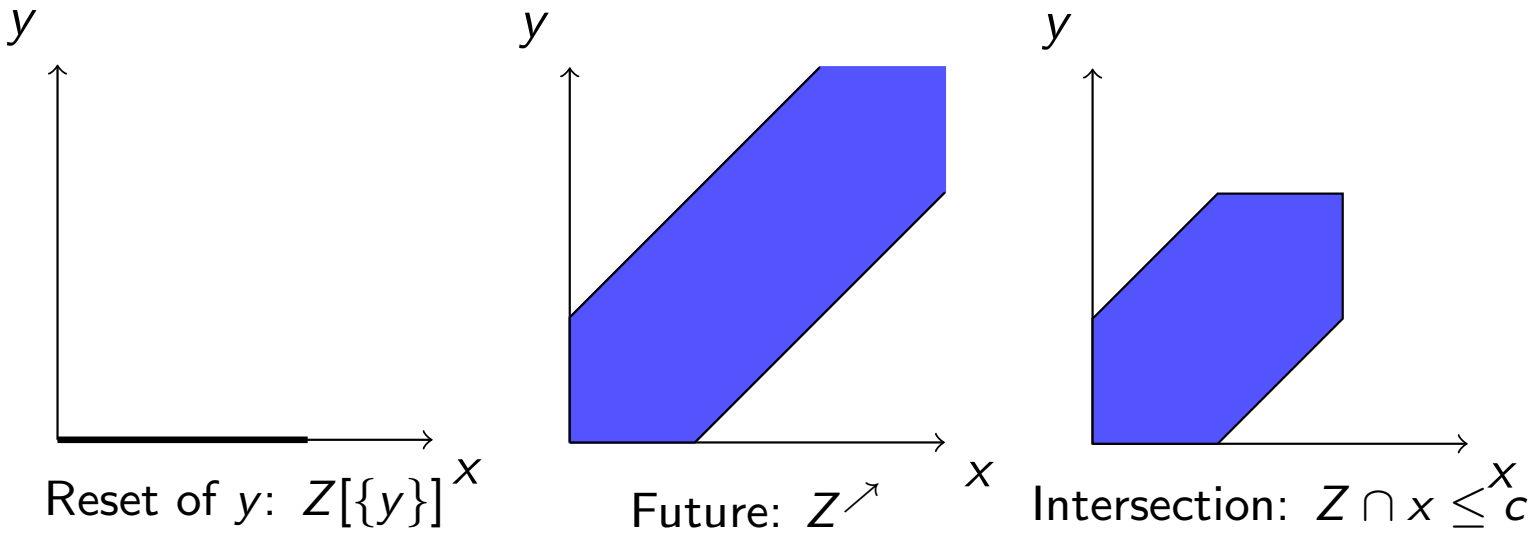

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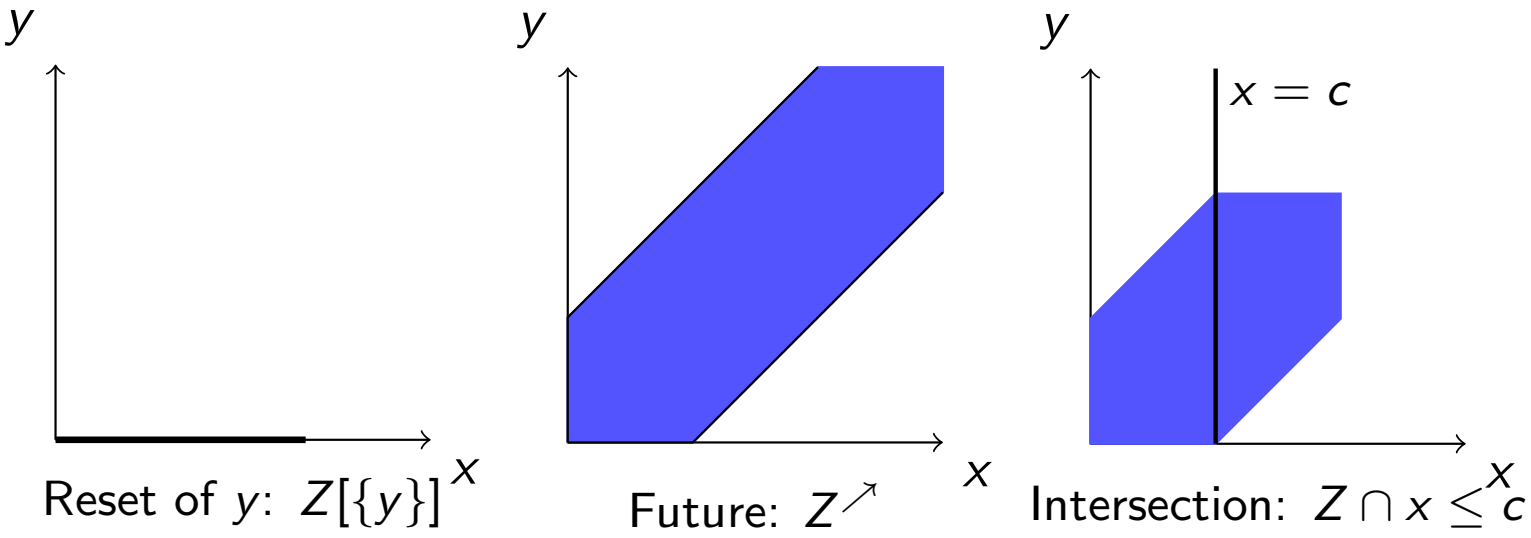

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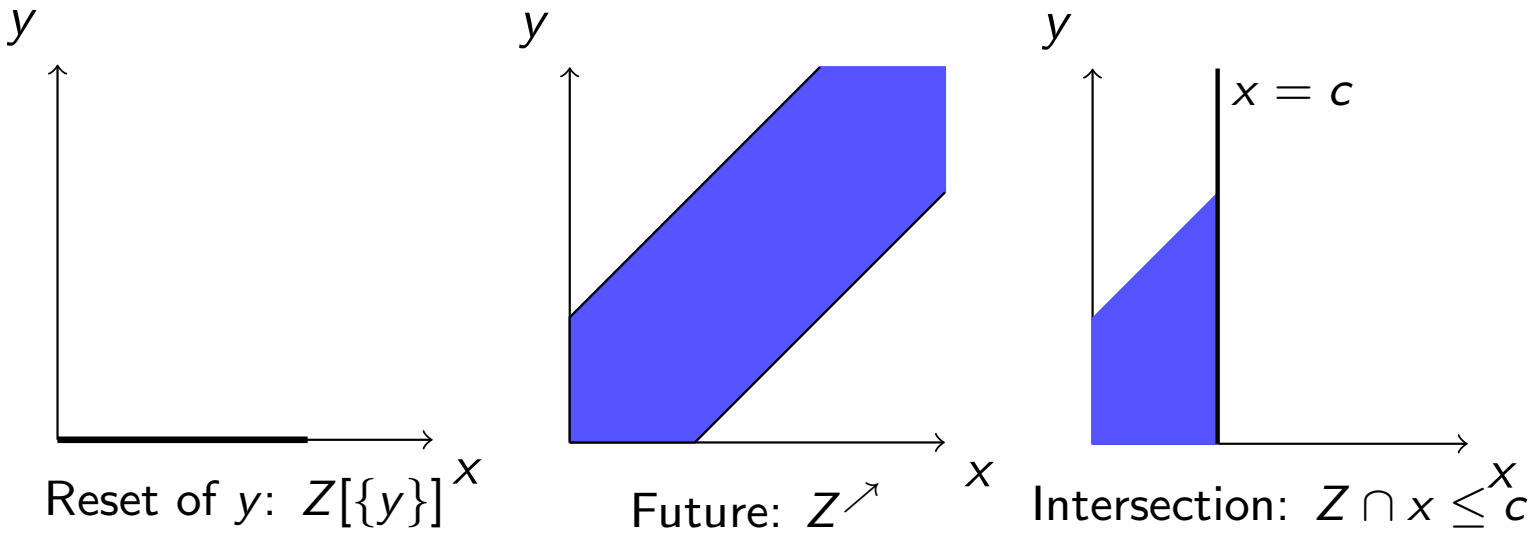

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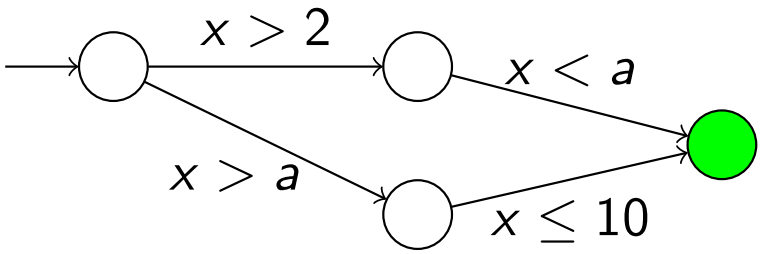

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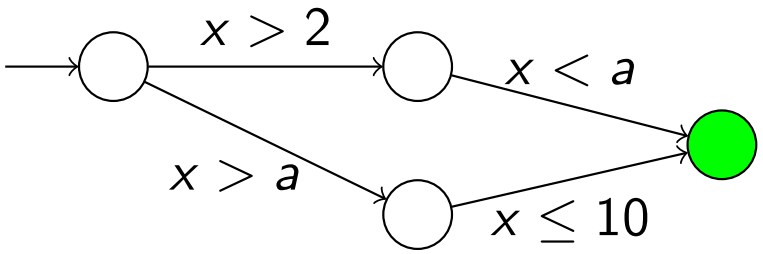

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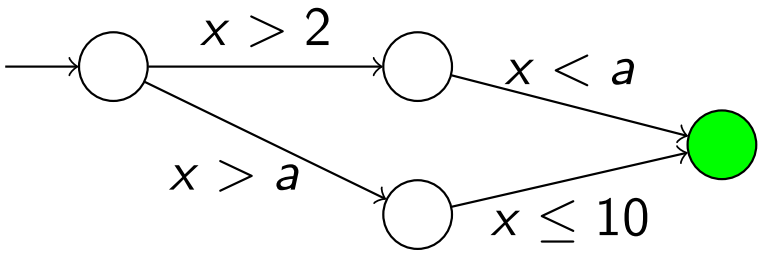

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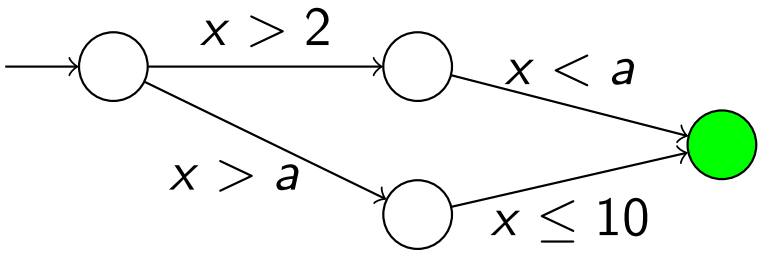

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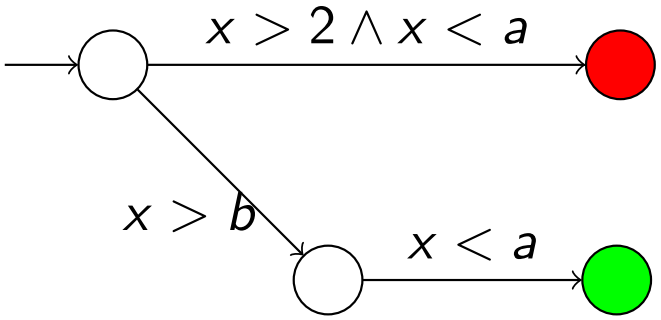

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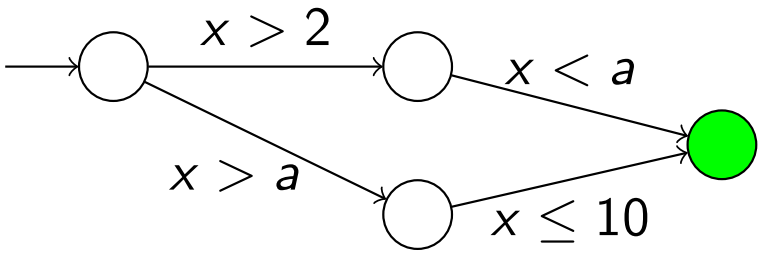

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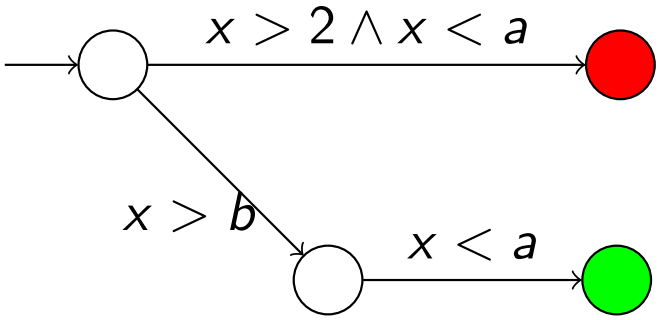

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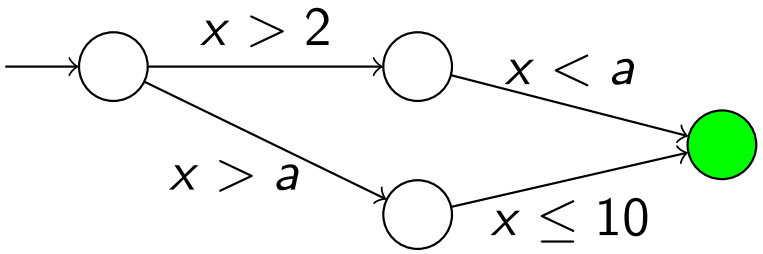

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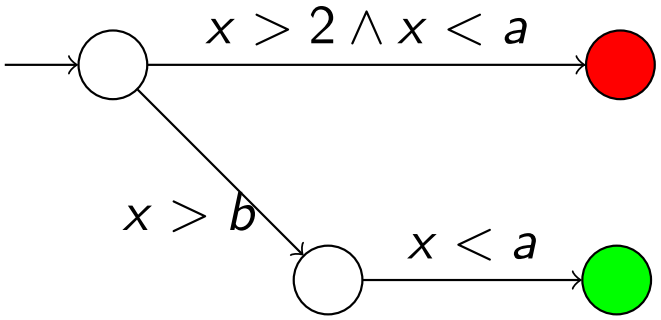

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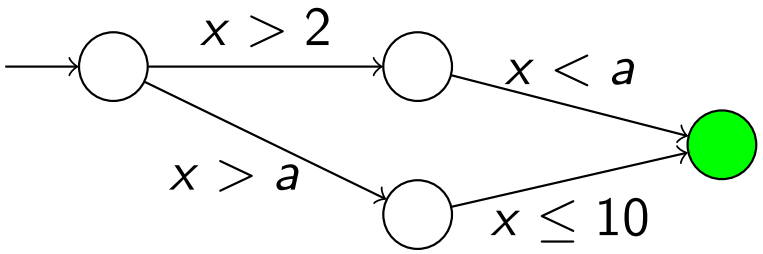

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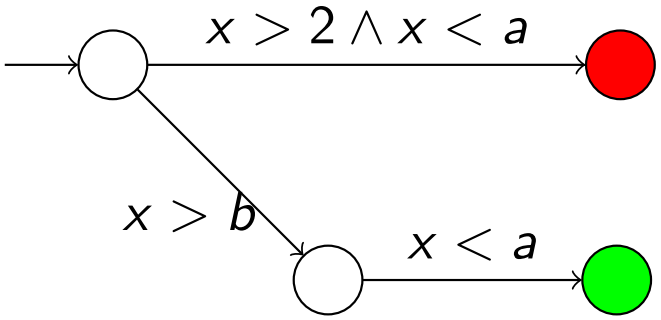

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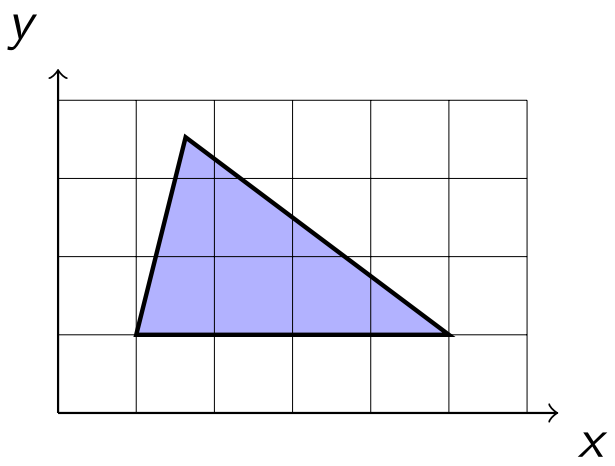
- Problem: **Termination** is **not** guaranteed.

Computing the Integer Valuations: Integer Hulls

- We compute **integer hulls**: $\mathrm{IntHull}(Z) = Conv(IntVects(Z))$



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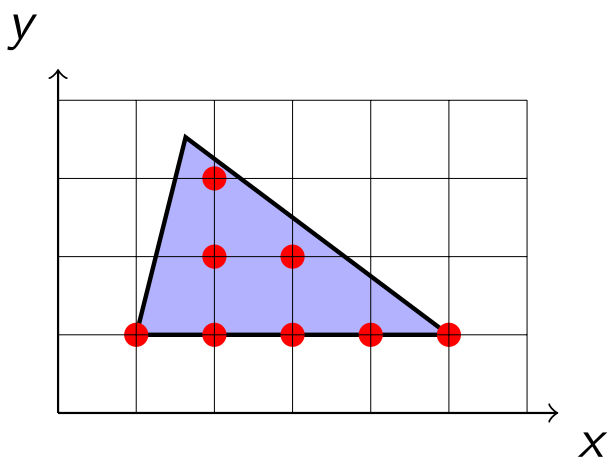

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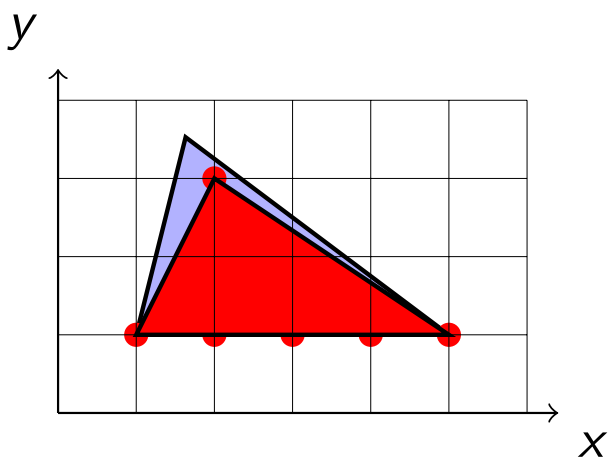

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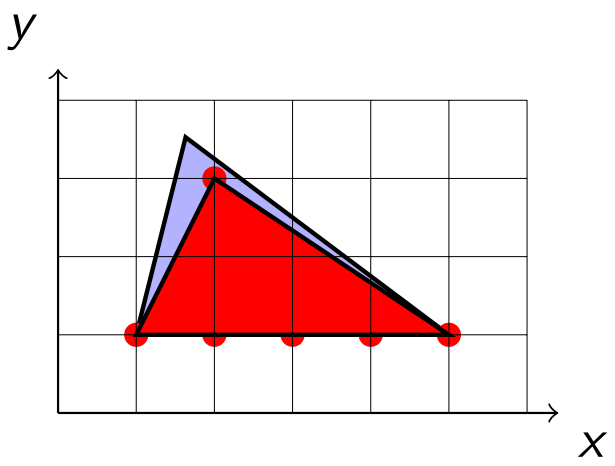

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Theorem [Soundness and completeness] [JLR13a]

For any PTA $\mathcal{A}$ and any subset of its locations G, upon termination, $\mathsf{IEF}(\mathsf{Init}(\mathcal{A}), G, \emptyset)$ (resp. $\mathsf{IAF}(\mathsf{Init}(\mathcal{A}), G, \emptyset)$) is the solution the integer EF-synthesis problem (resp. AF-synthesis problem) for PTA $\mathcal{A}$ and set of locations to reach G.

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- Termination?

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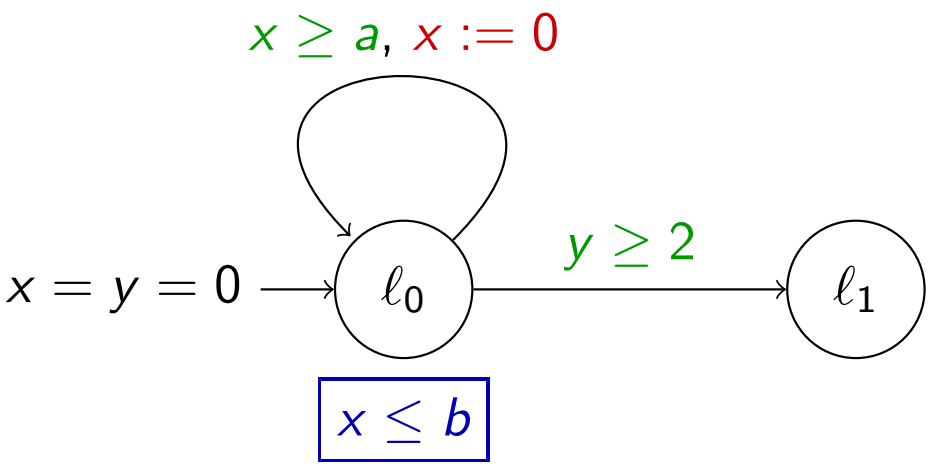

- Initial symbolic state: (ℓ_0, Z_0) with $Z_0 = \{x = y, x \leq b\}$;
- After n loops: (ℓ_0, Z_n) with **still**

$$Z_n = \{0 \leq x \leq b, 0 \leq na \leq y - x \leq (n+1)b\}$$

- Actually, $Z_n = \mathrm{IntHull}(Z_n)$

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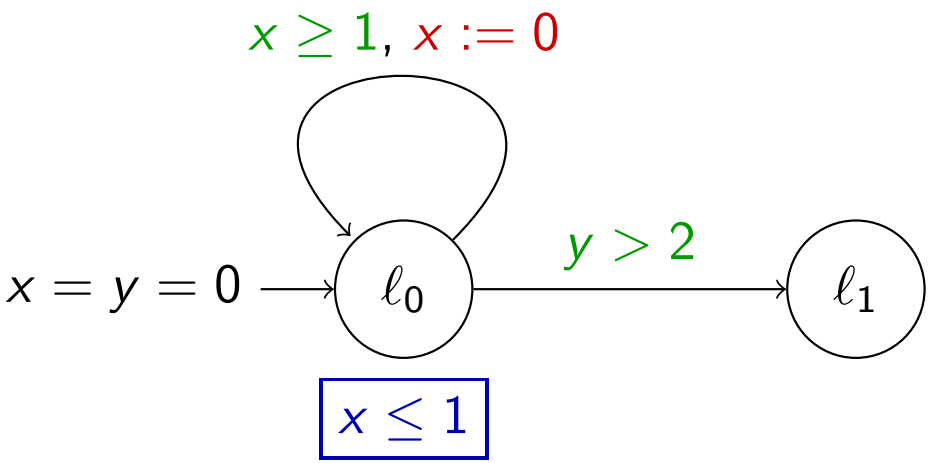

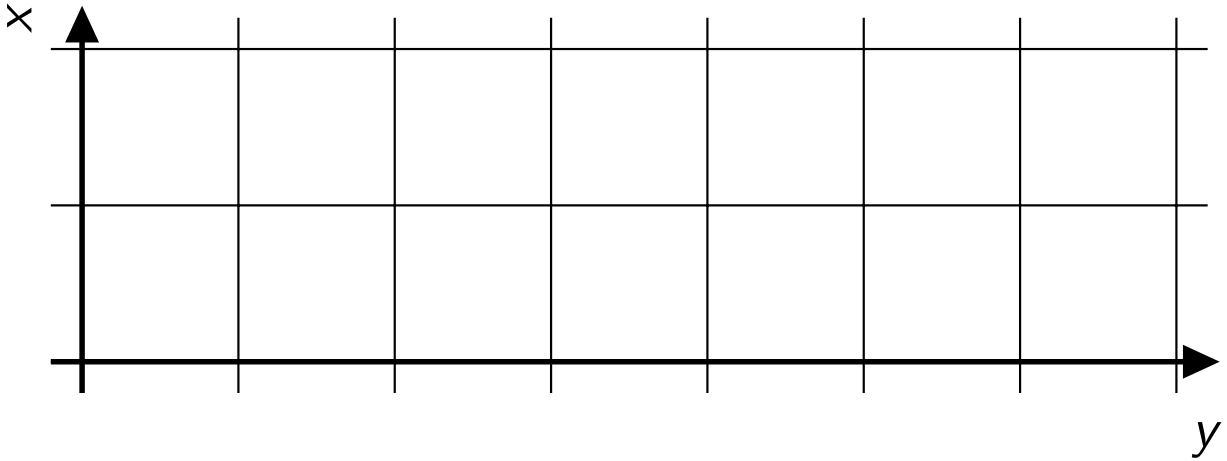

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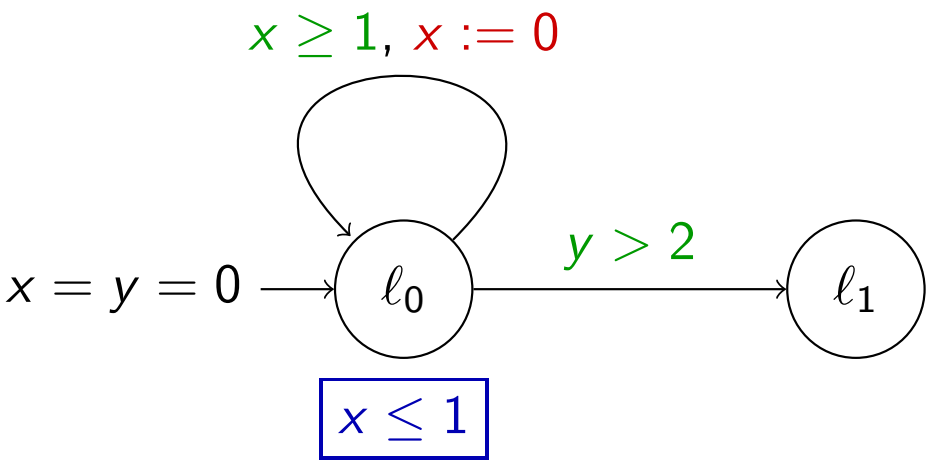

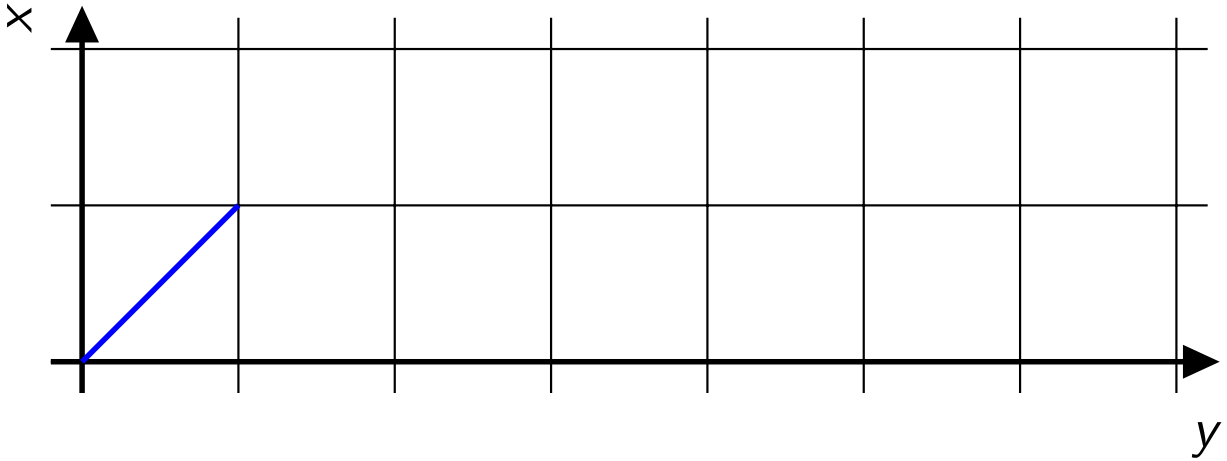

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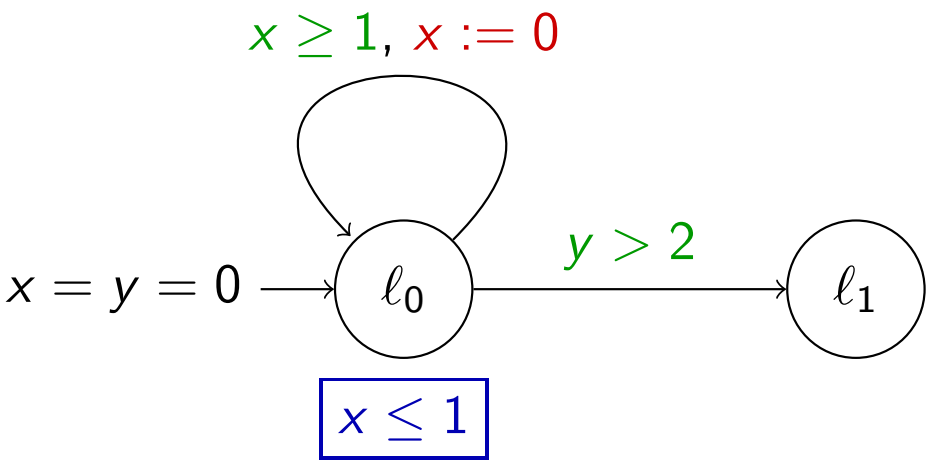

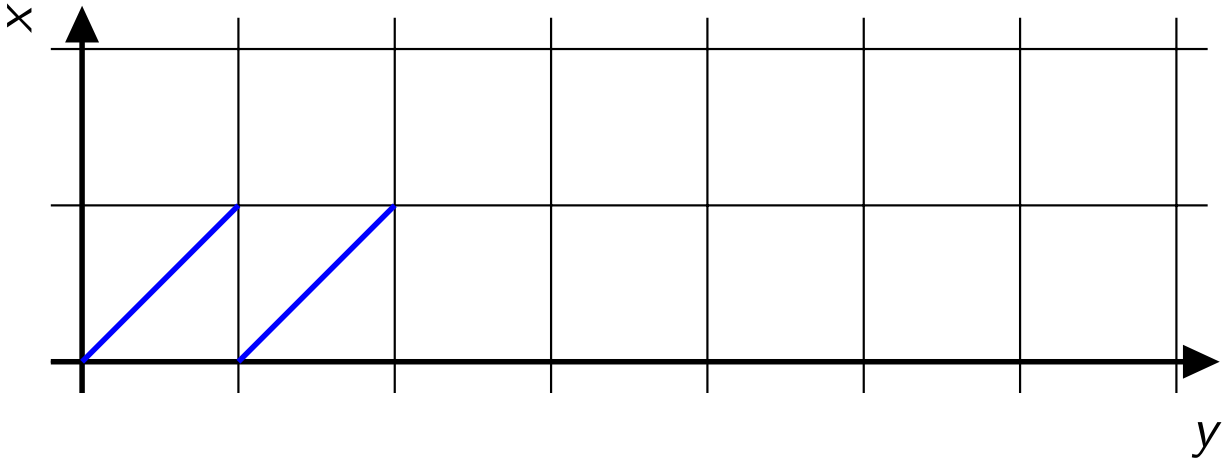

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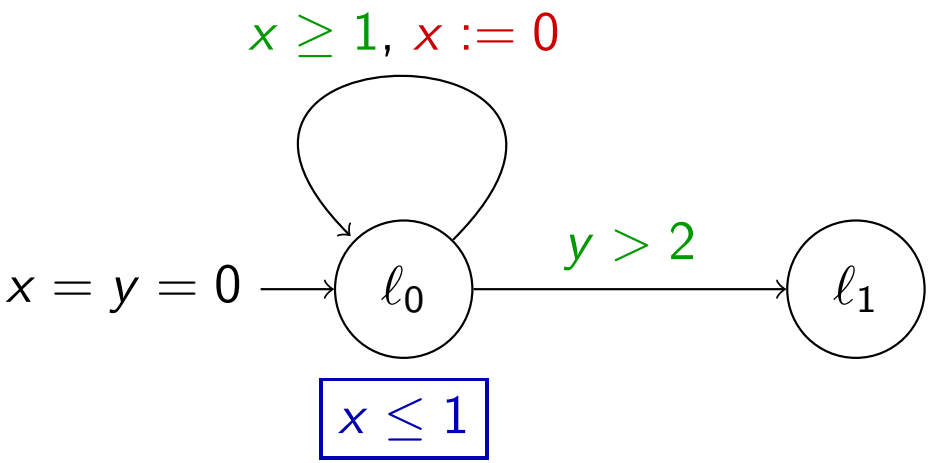

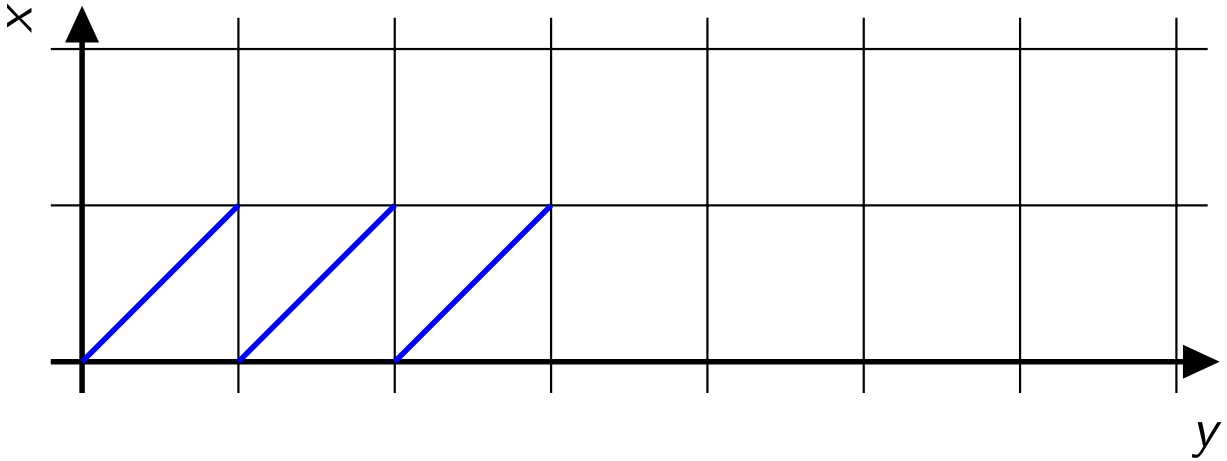

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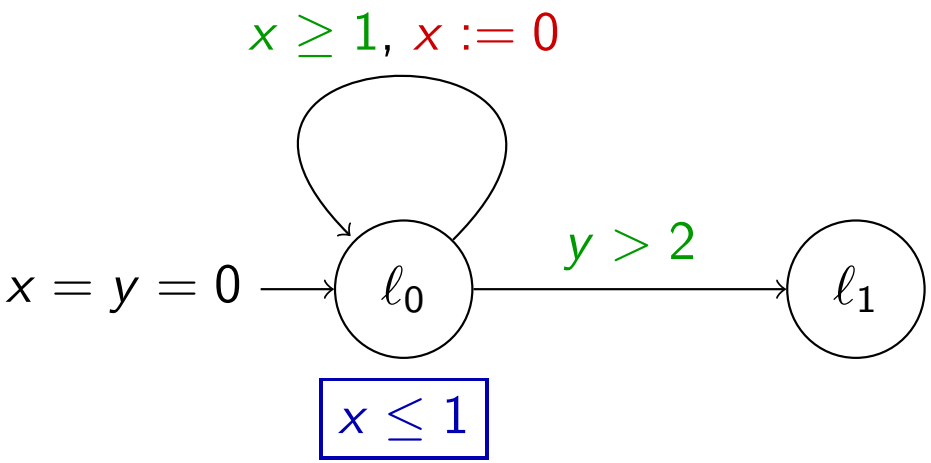

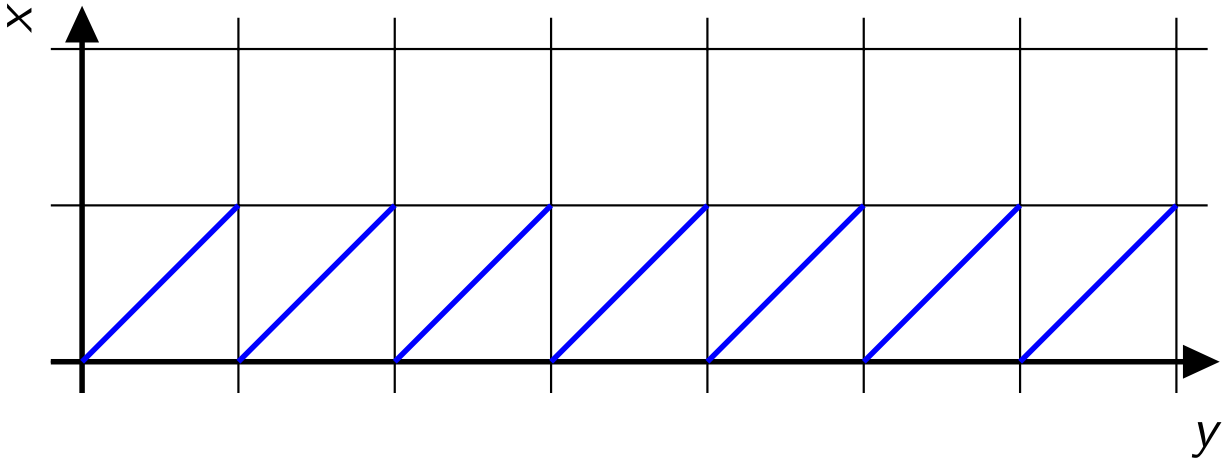

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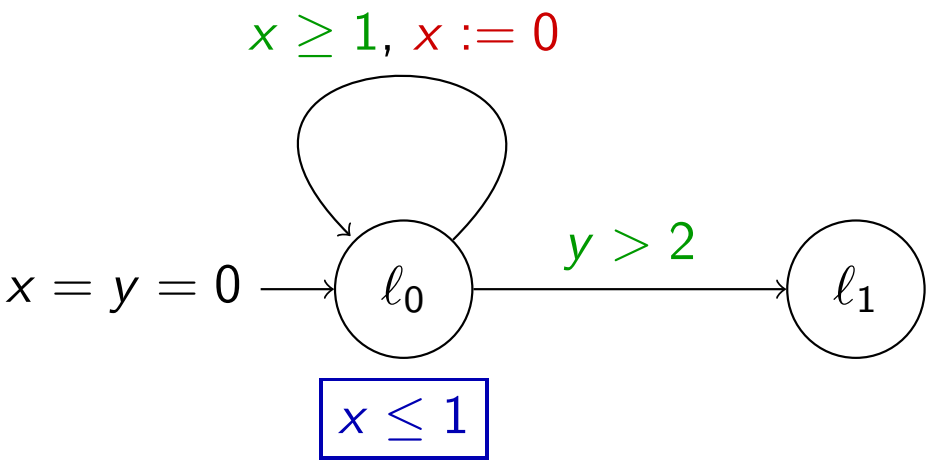

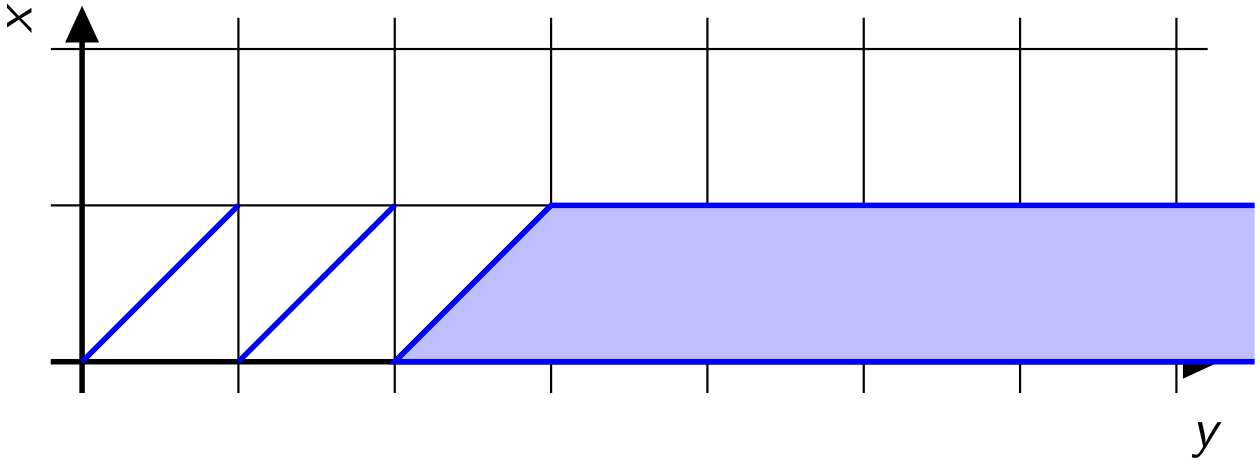

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- Let K be the maximal constant appearing in PTA, $R(K)$ the number of regions and $|L|$ the number of locations

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Termination in the Bounded Case

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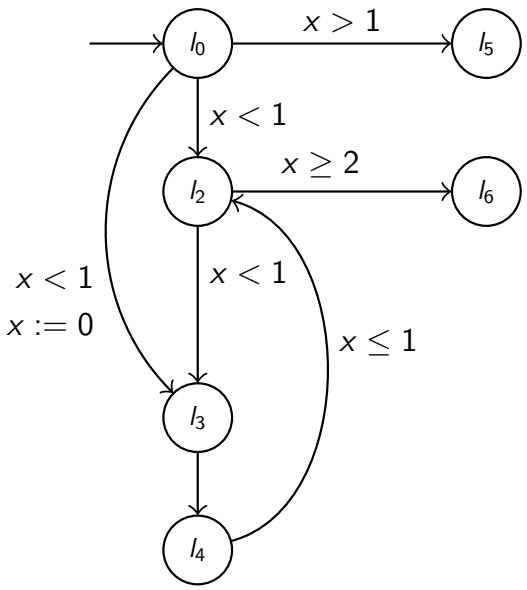
- Reachability and unavoidability are preserved.
- **Finite** number of zones $\implies$ **termination** guaranteed.

Outline

Real-Timed Control: Timed Game Automata [MPS95]

Control problem:

- system is modeled as a timed automaton



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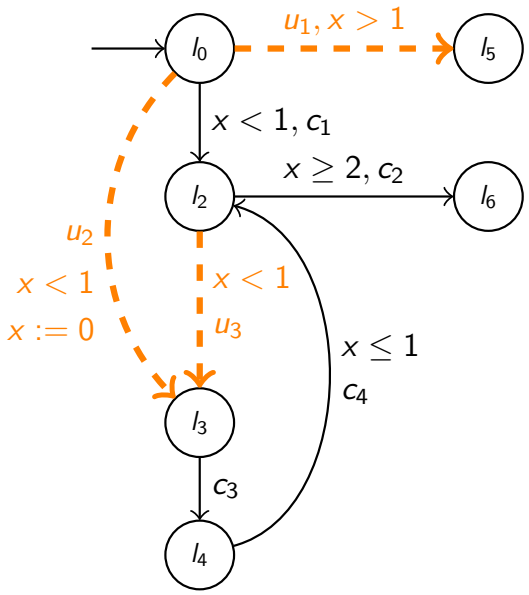

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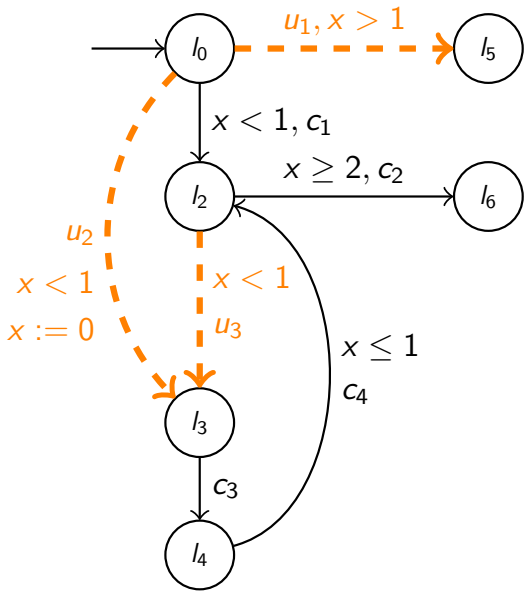

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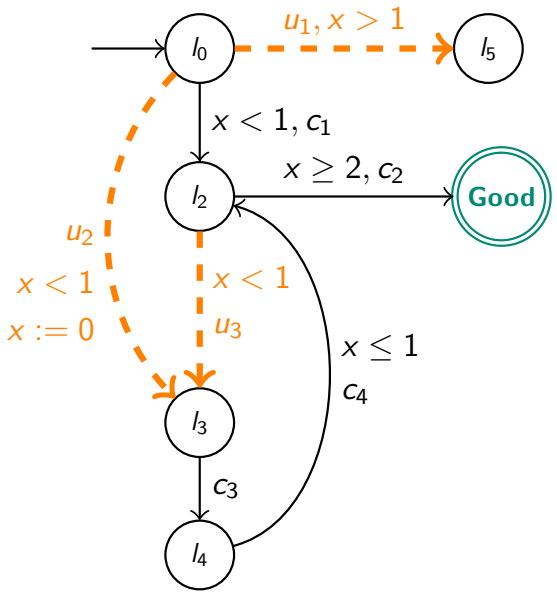

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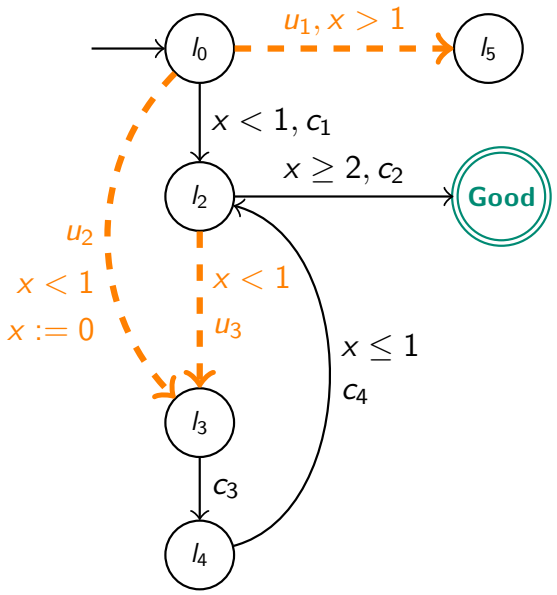
Controller synthesis:

- find a **winning strategy** to reach Good



Parametric Timed Games

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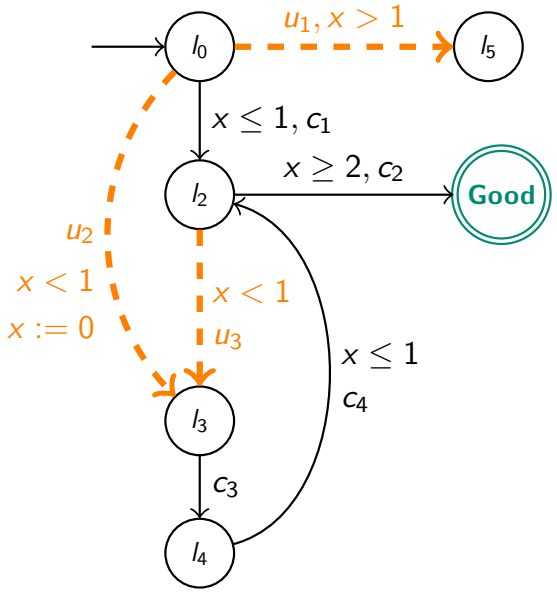

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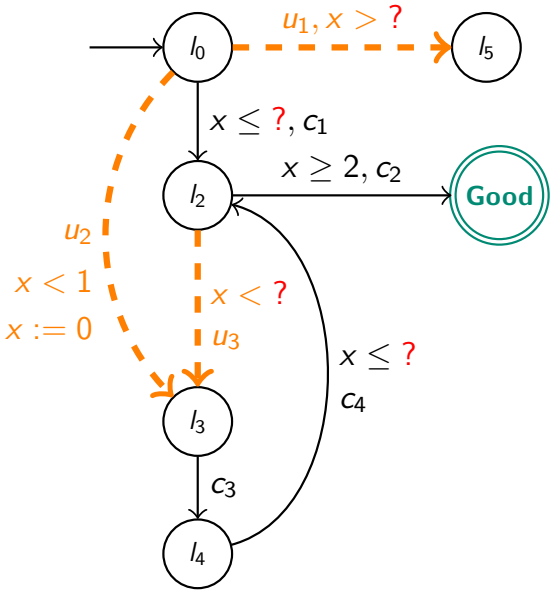

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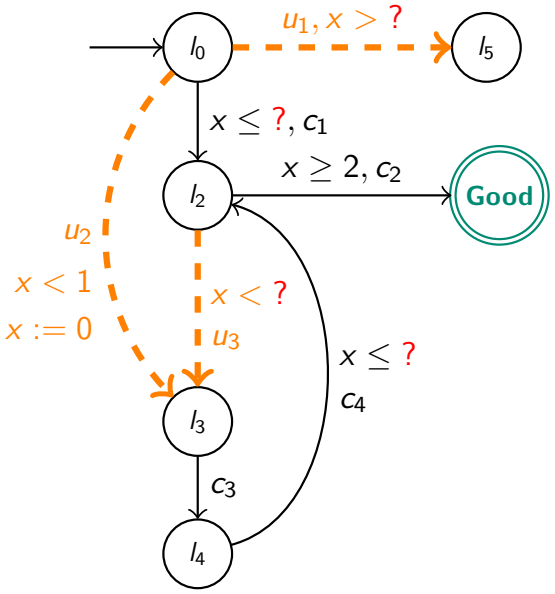

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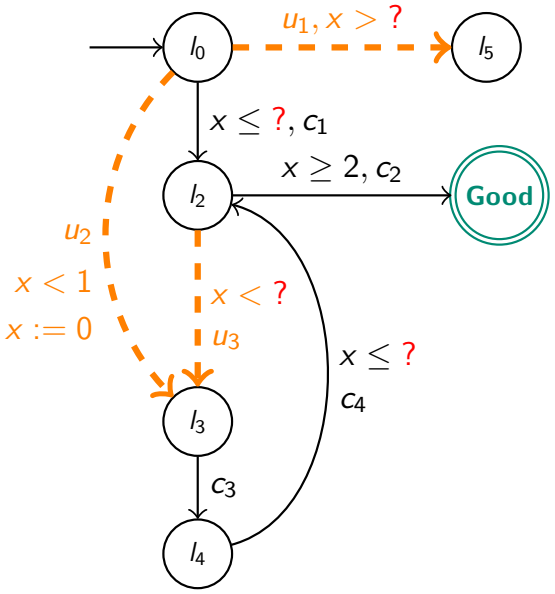

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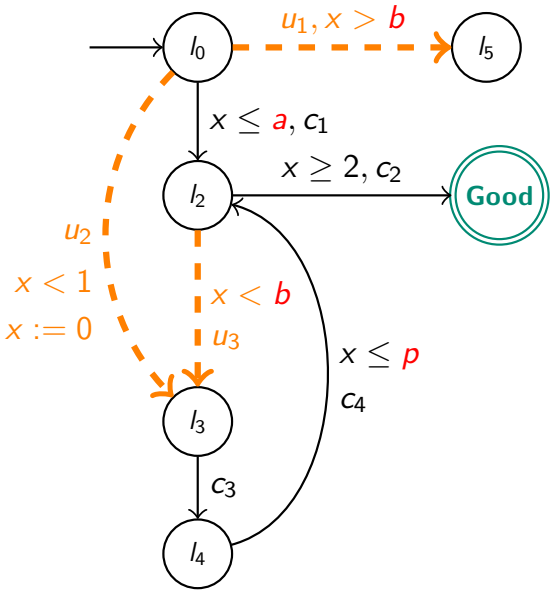
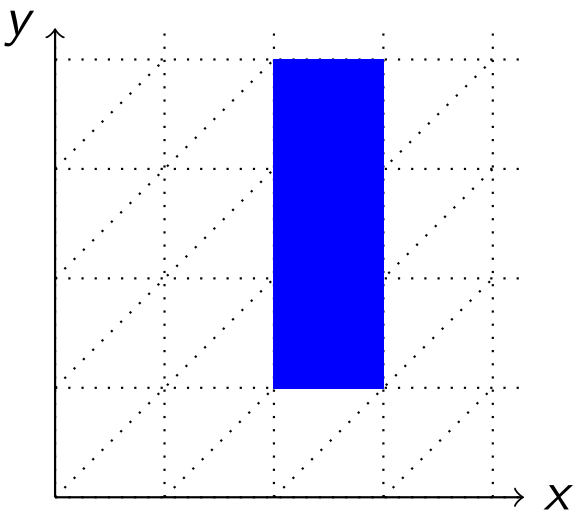
Parametric model:

- **Parametric timed game automata** (PGA) [JFLR12]
- *control problem* on PGA: compute parameter valuations s.t. there exists the winning strategy.



Backward Computation of Winning States in PGA [MPS95]



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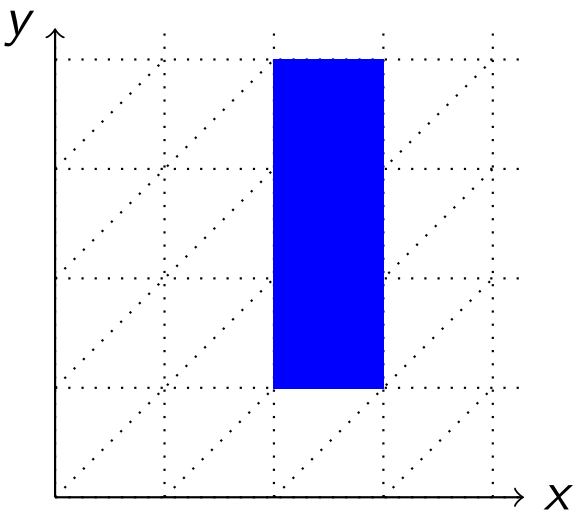

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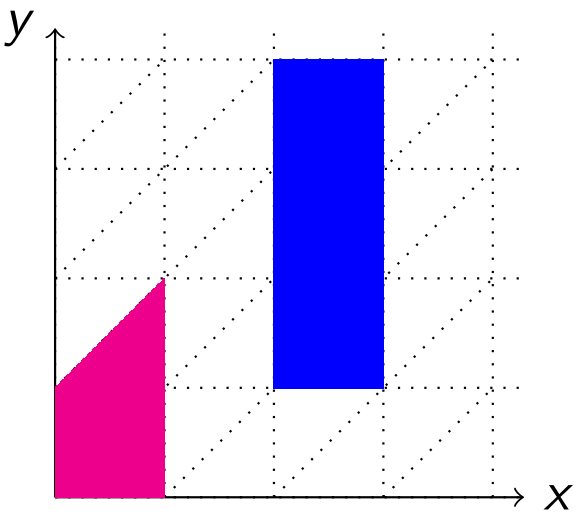

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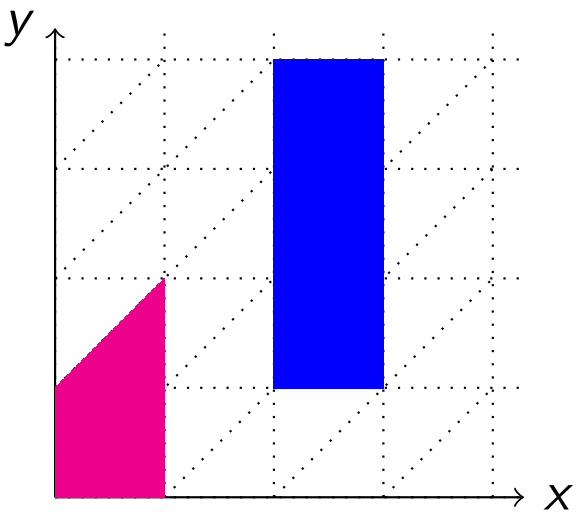

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Safe-timed predecessors:

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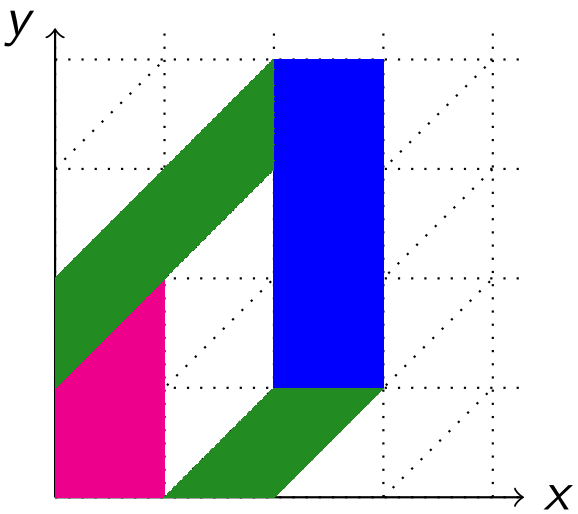

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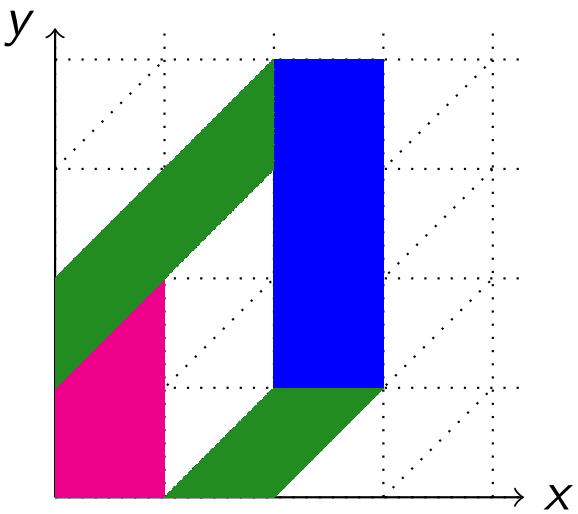

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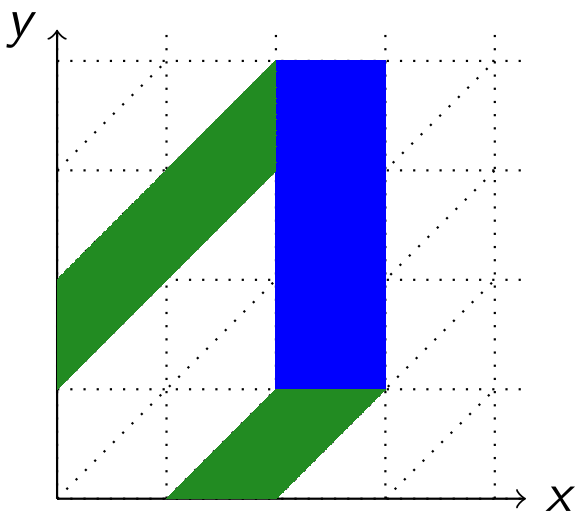
Computation of the winning states: *controllable predecessors*

$$\mathrm{Pred_t}(X \cup \mathrm{cPred}(X), \mathrm{uPred}(\neg X))$$

Synthesis of Integer Parameters: Integer Shape

Problem: in the parametric domain the computation might not terminate.



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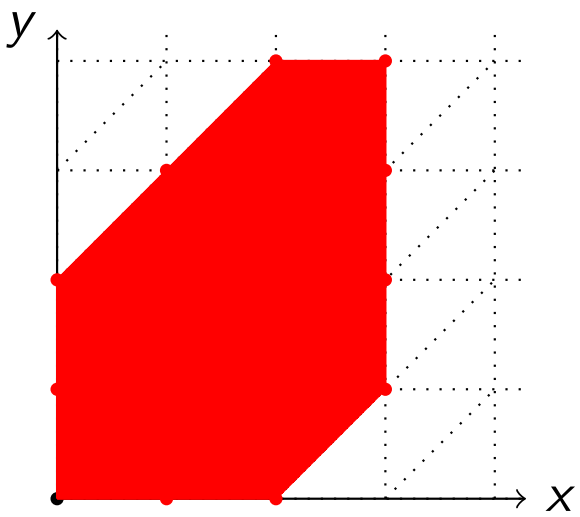

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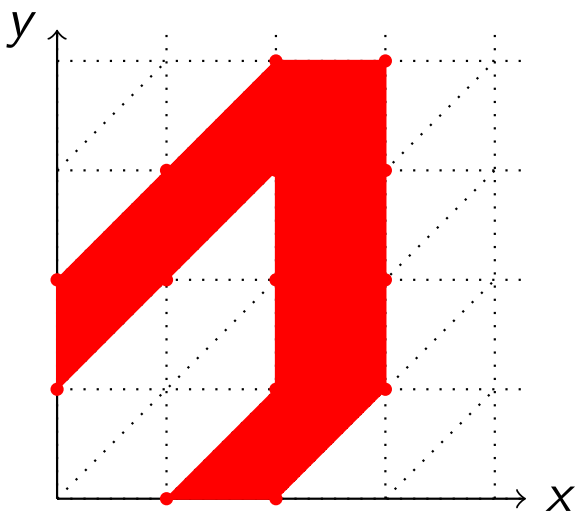

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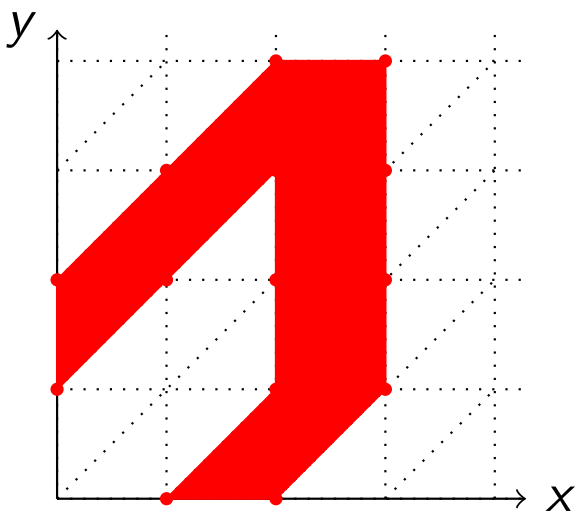

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- Our algorithm: bounded integer computation (forward: IntHull, backward: IntShape) [JLR13b].

Example

Backward computation starts from:
$(Goal, x \geq 2)$.

The set of **winning states**:

- $(\ell_3, (x \geq 0 \wedge a \geq b))$,
- $(\ell_2, (x \geq b) \vee (x \geq 0 \wedge a \geq b))$,
- $(\ell_1, (x \leq a \wedge a \geq b))$,
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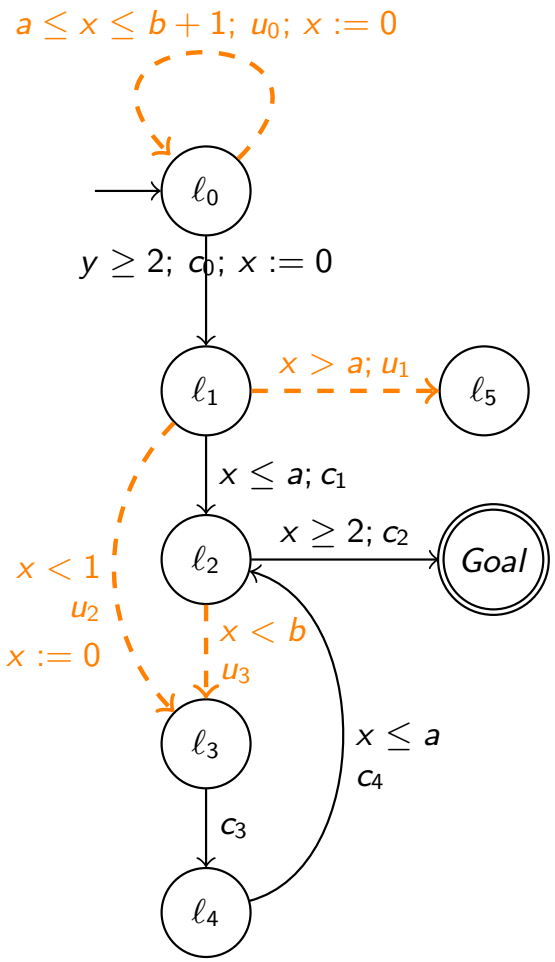

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The **set of constraints** obtained:

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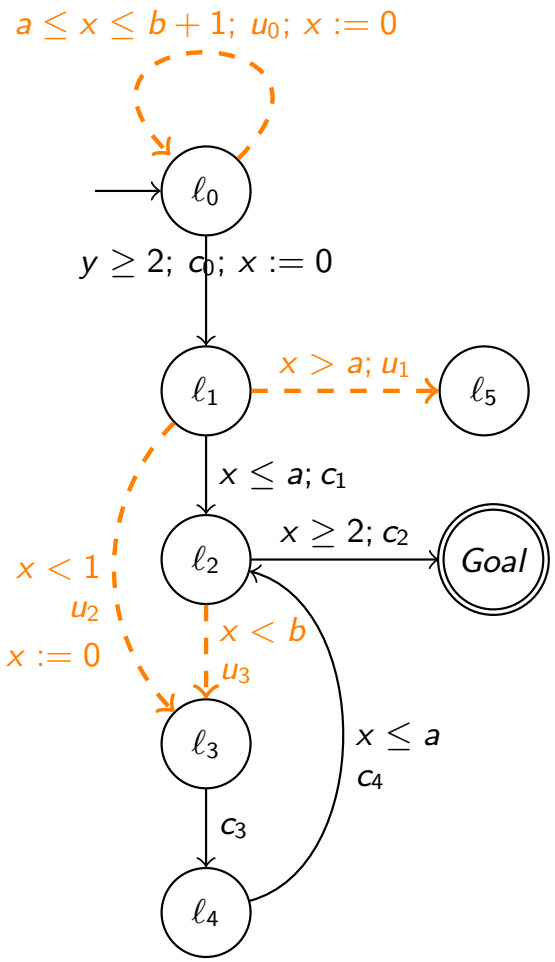

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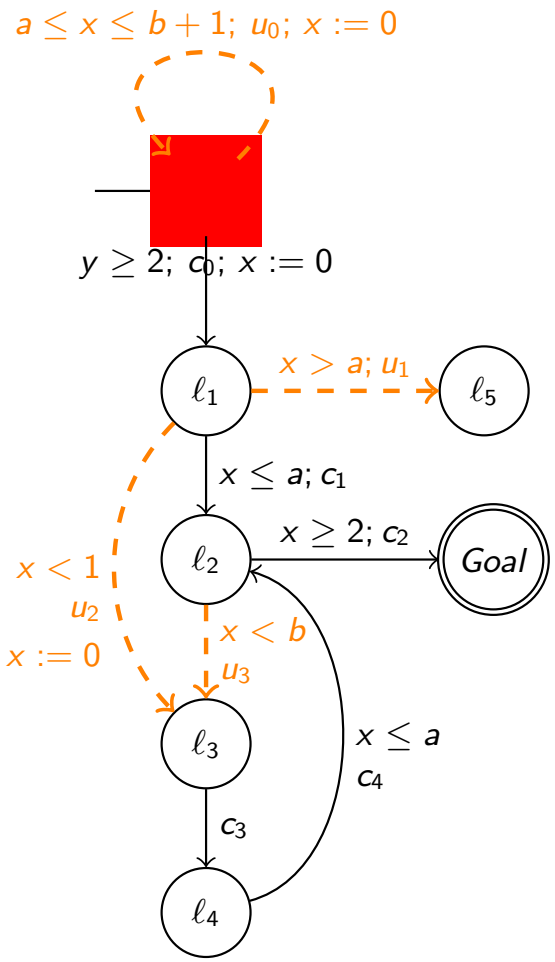

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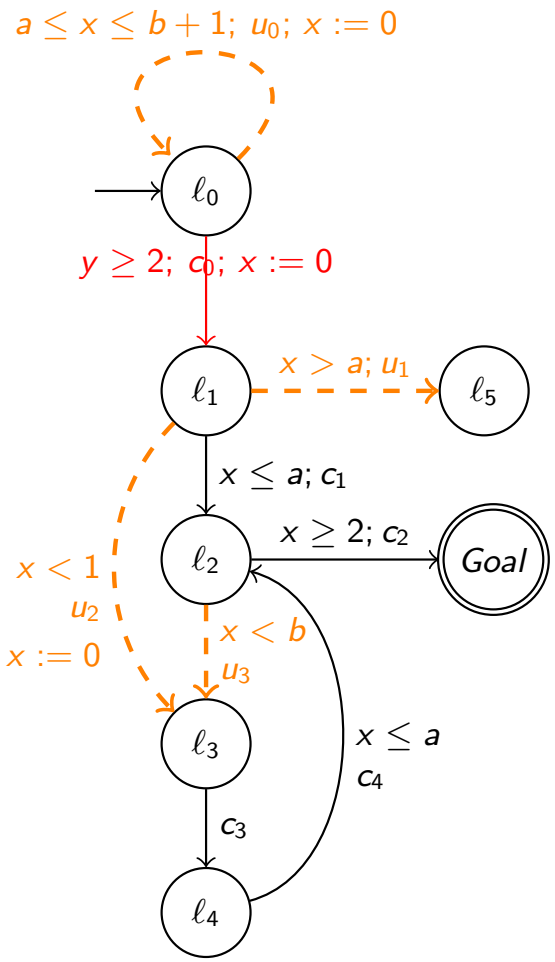

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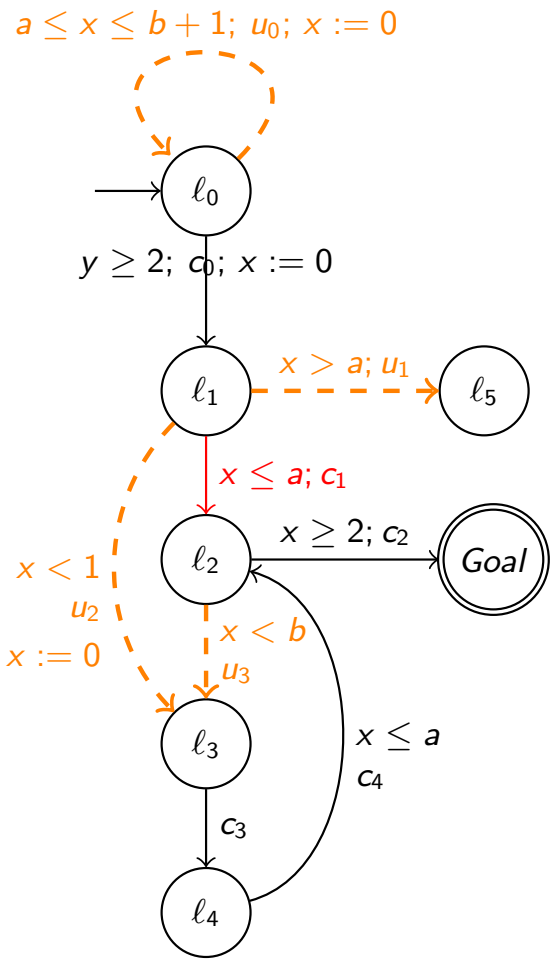

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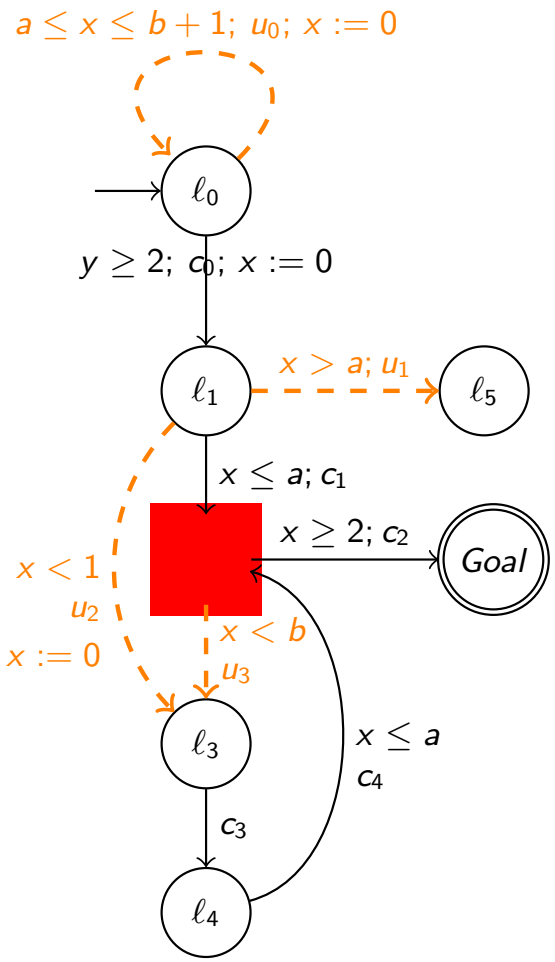

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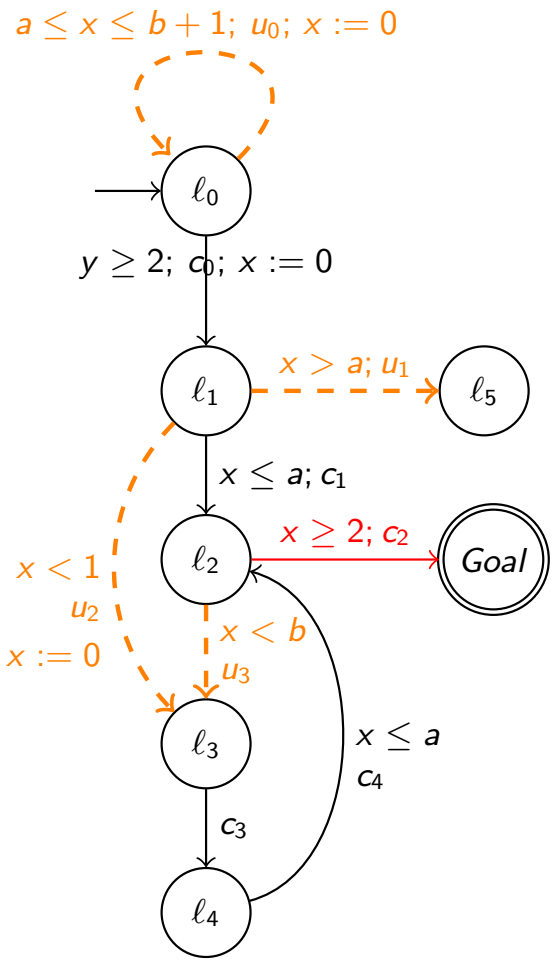

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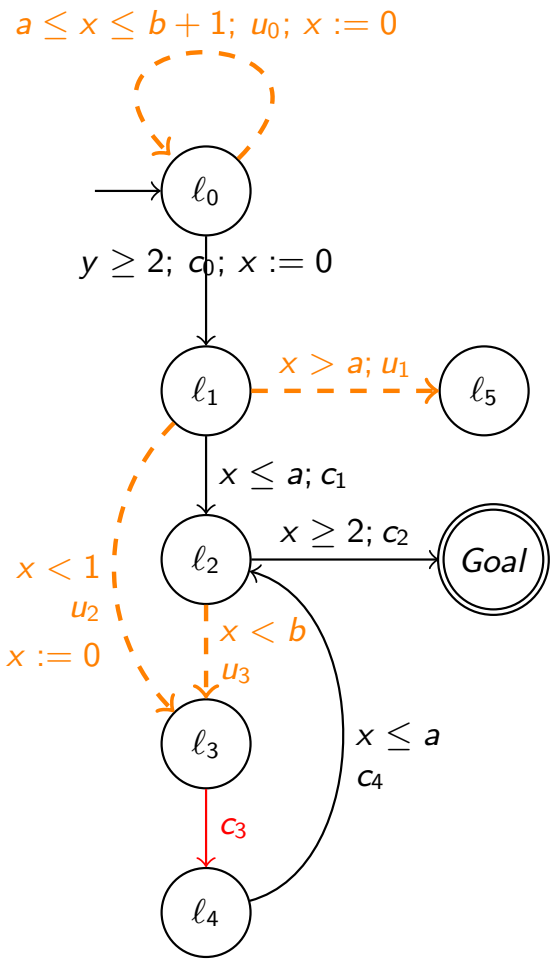

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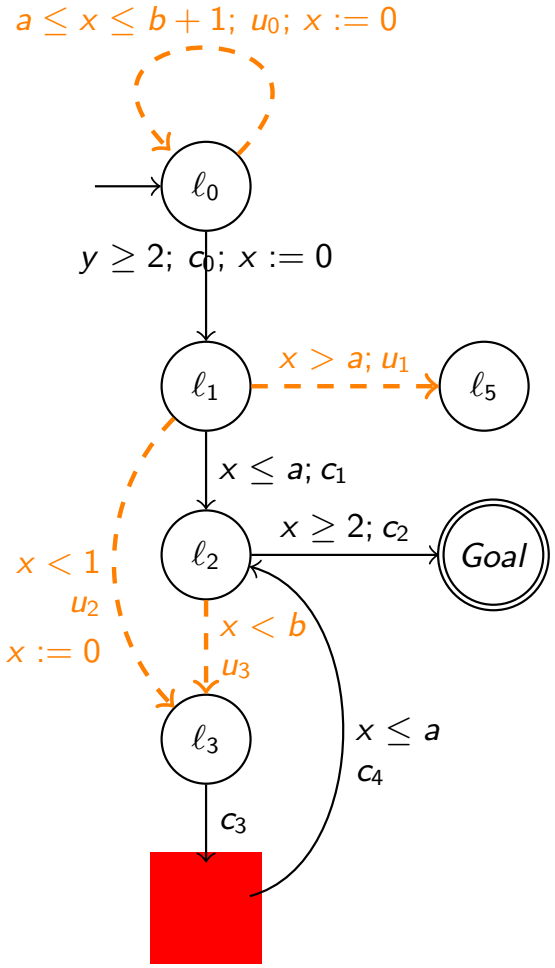

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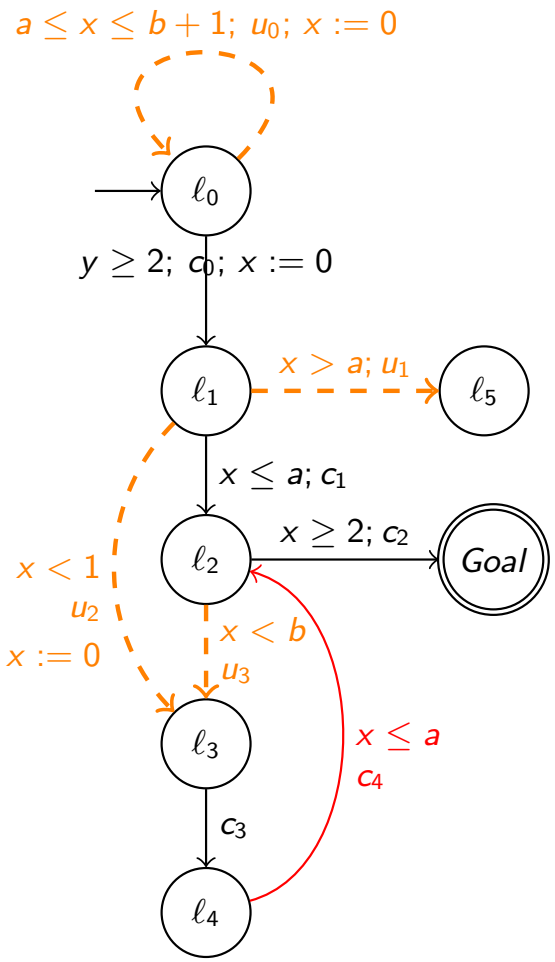

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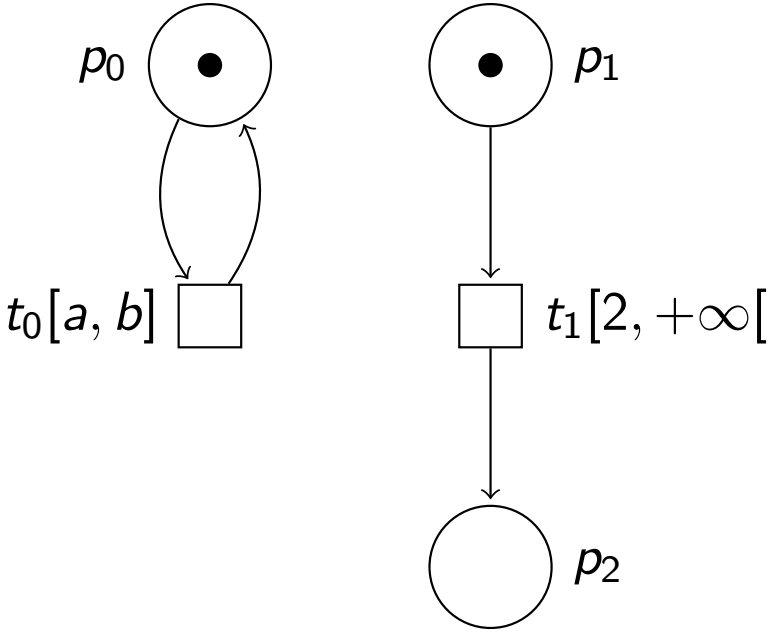
Outline

Parametric Time Petri Nets



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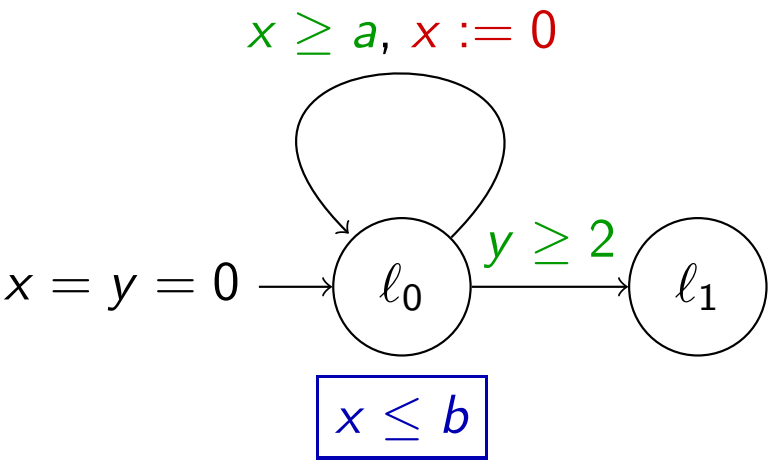
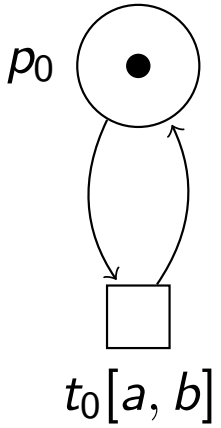
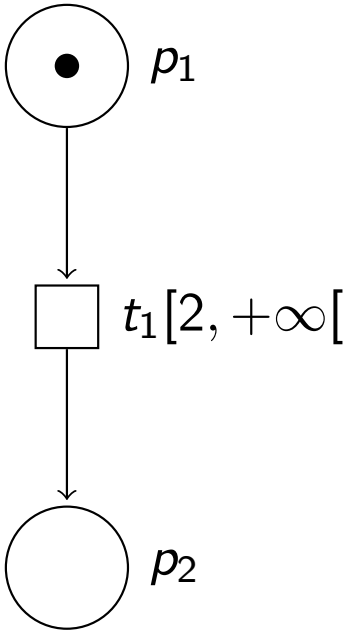
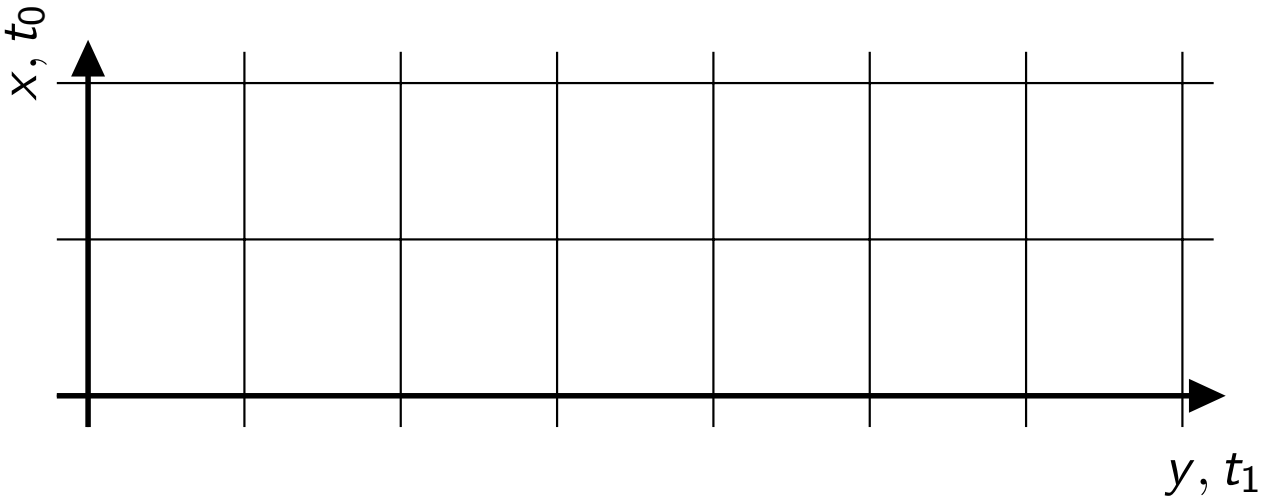
- In both cases, we can use the integer parameters approach.

Termination in the Bounded Case









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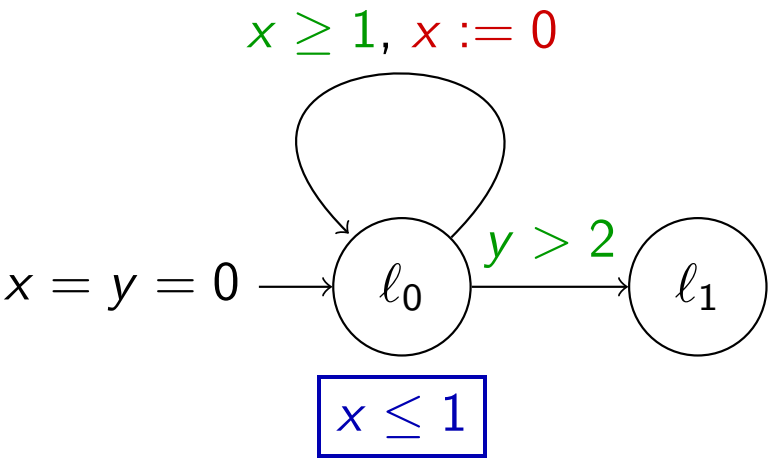

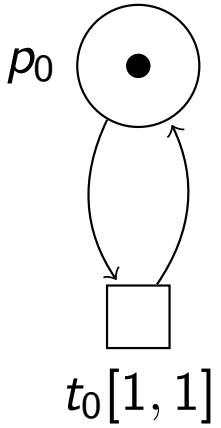

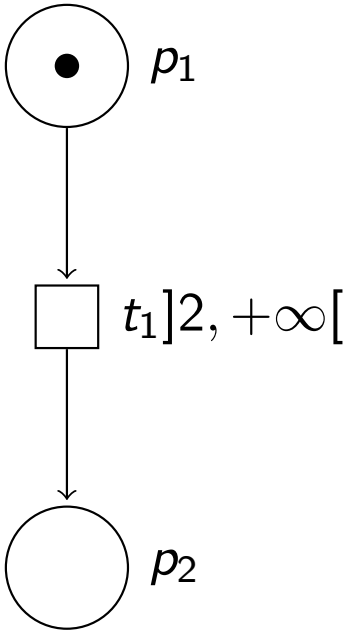

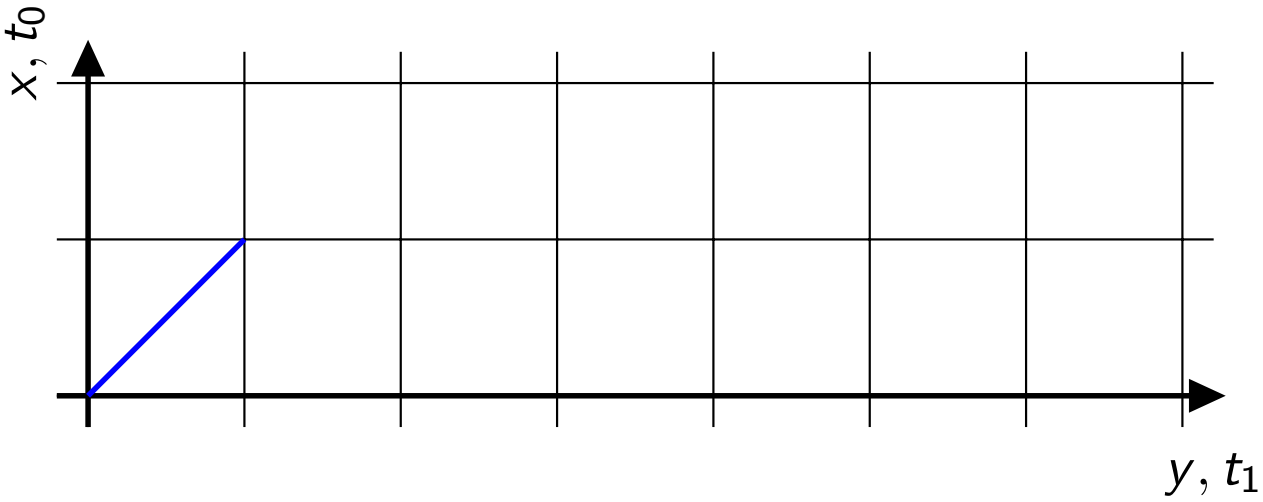

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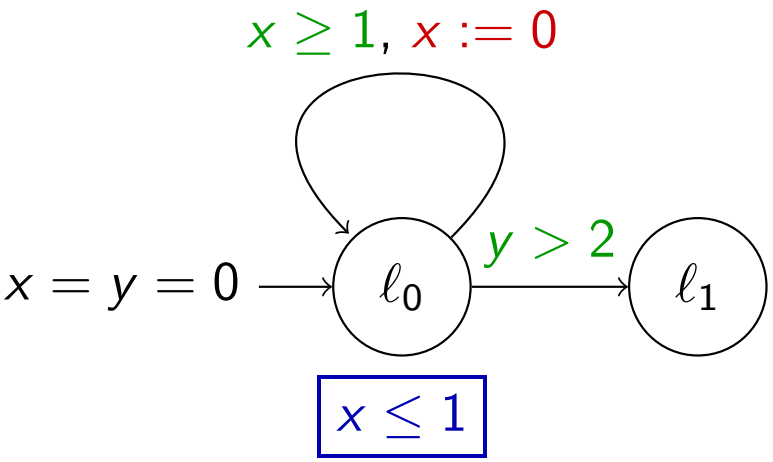

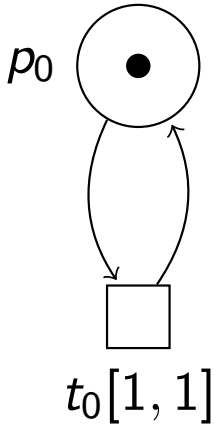

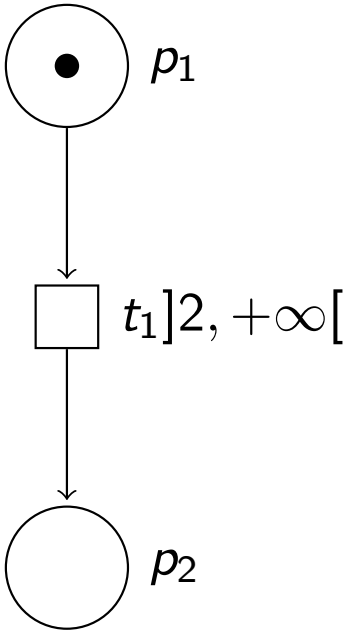

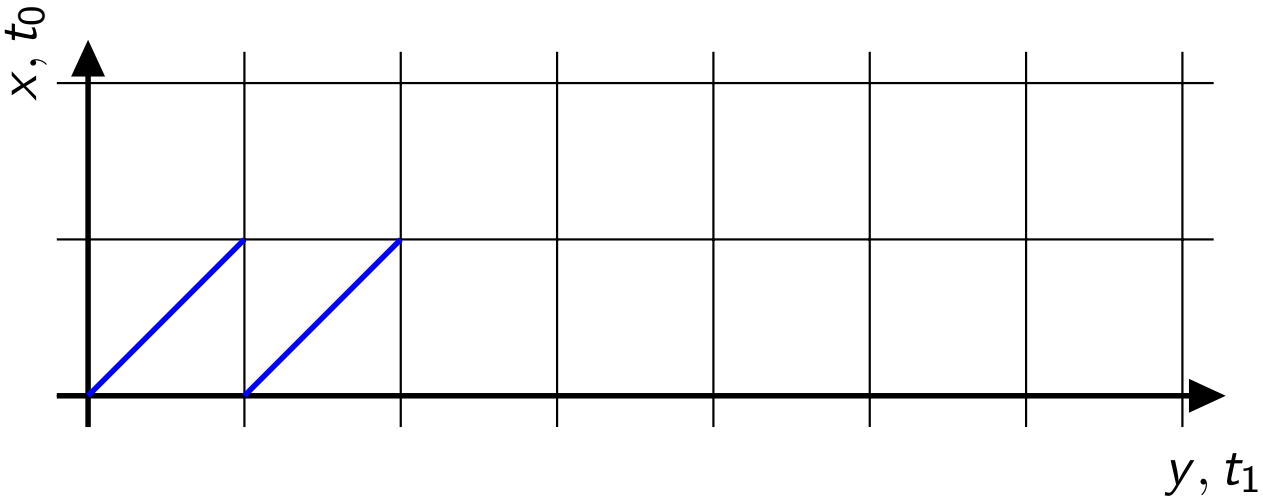

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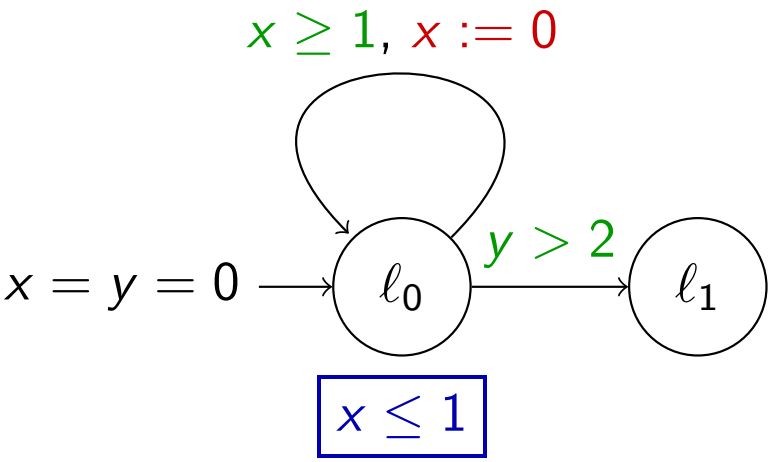

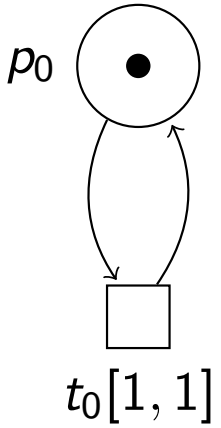

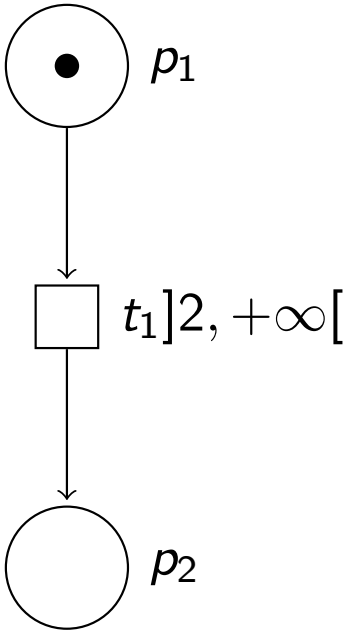

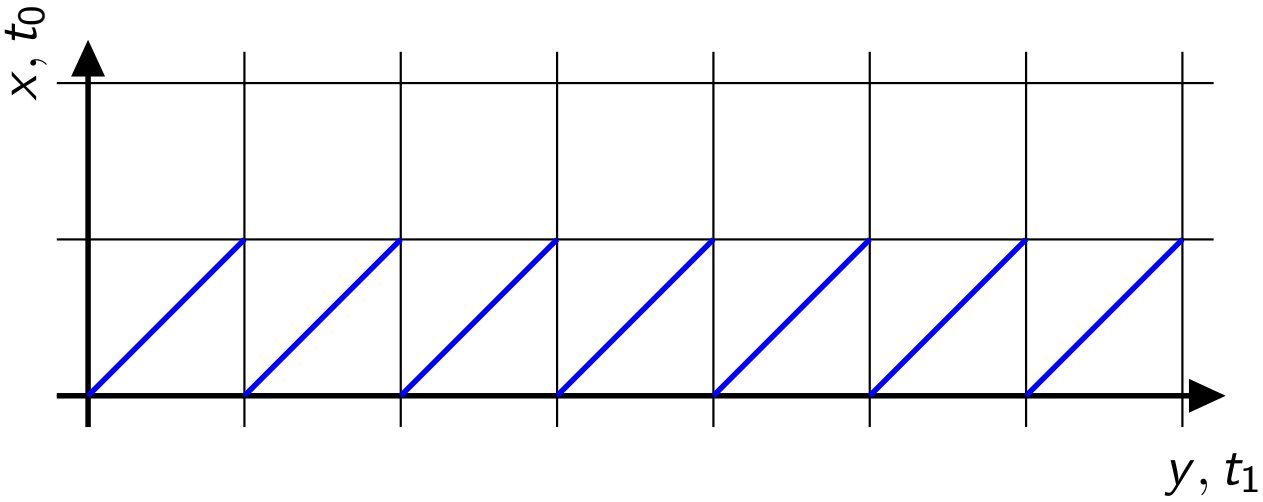

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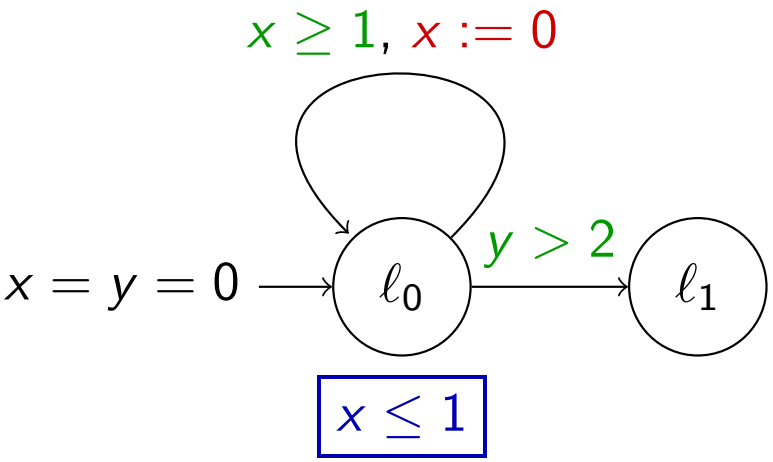

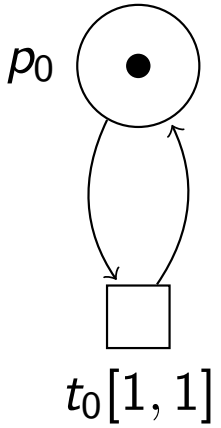

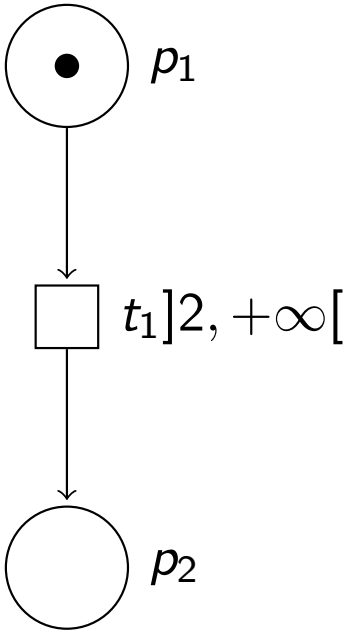

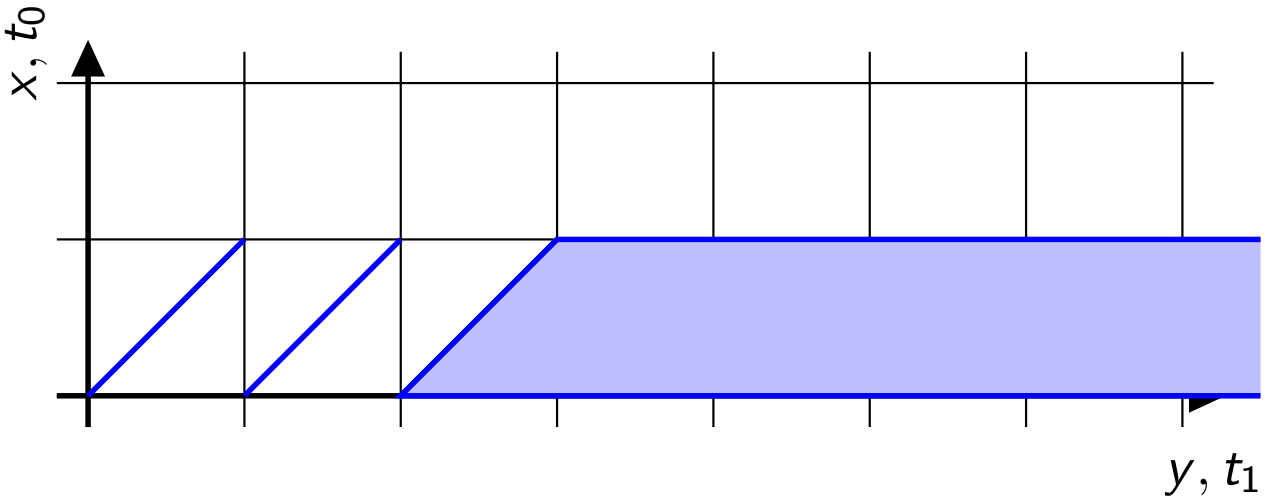

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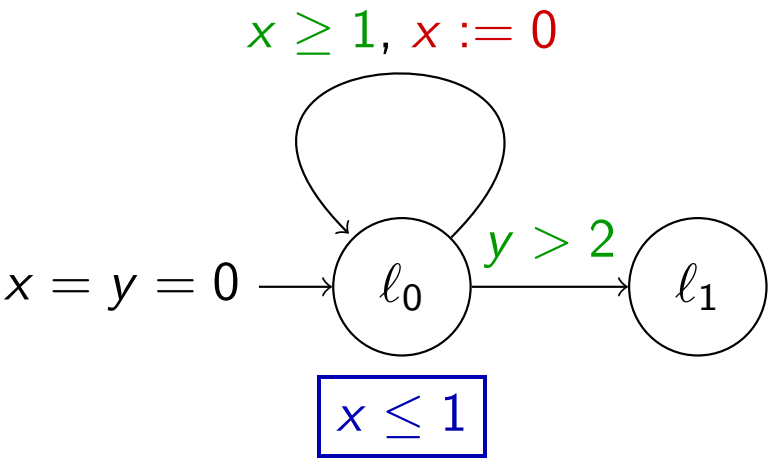

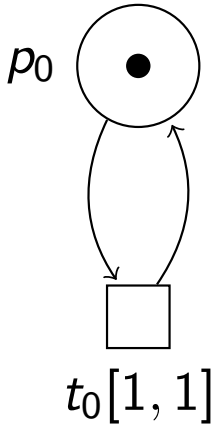

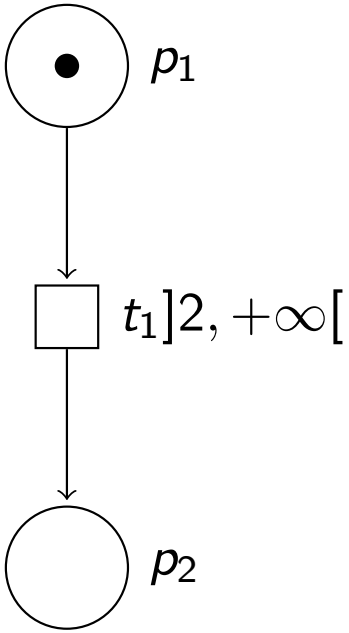

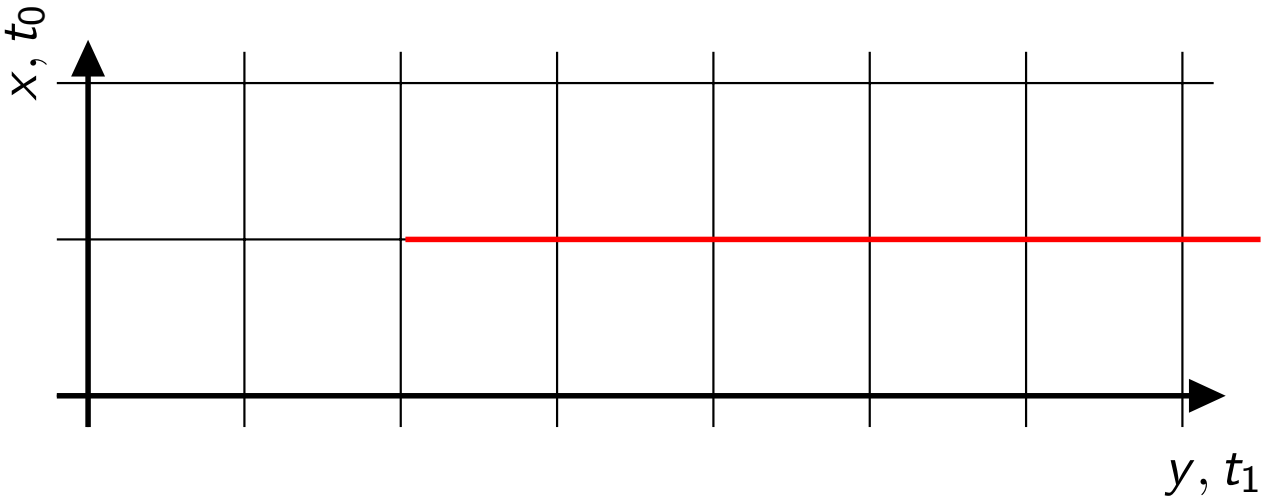

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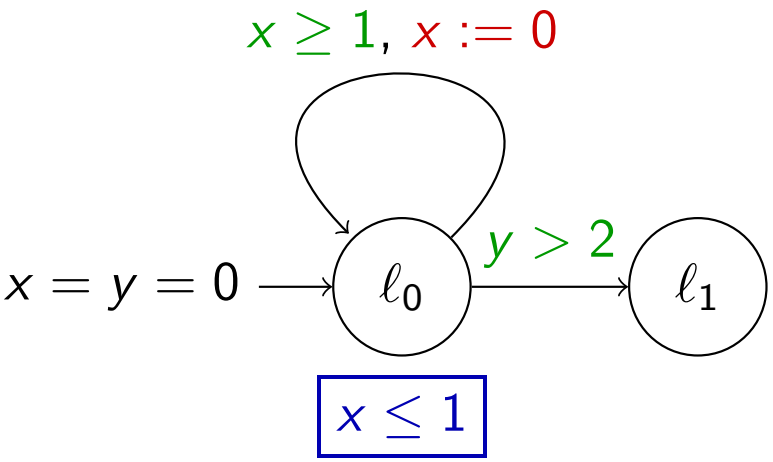

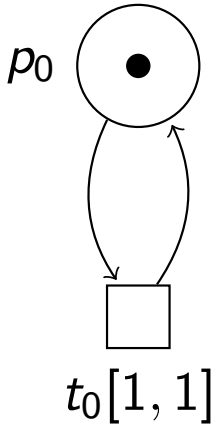

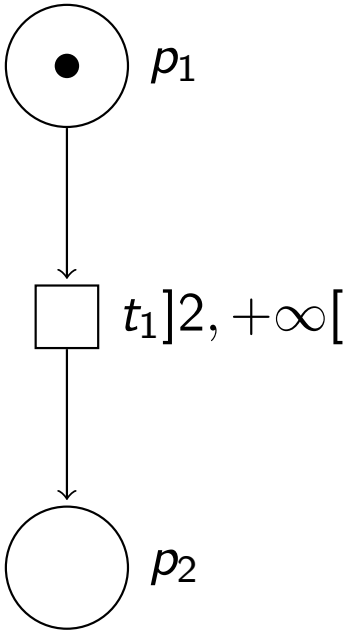

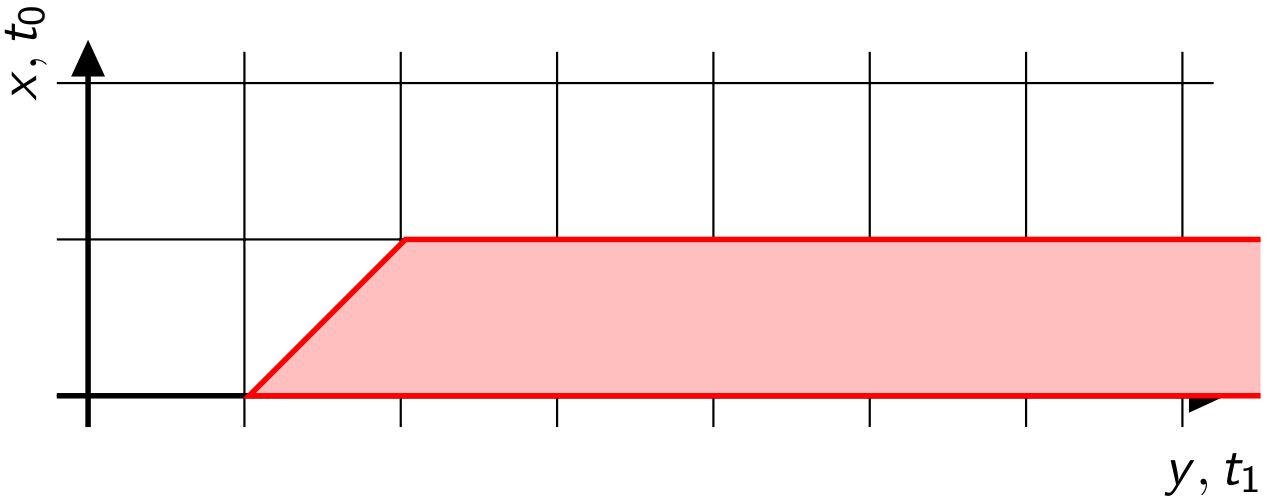

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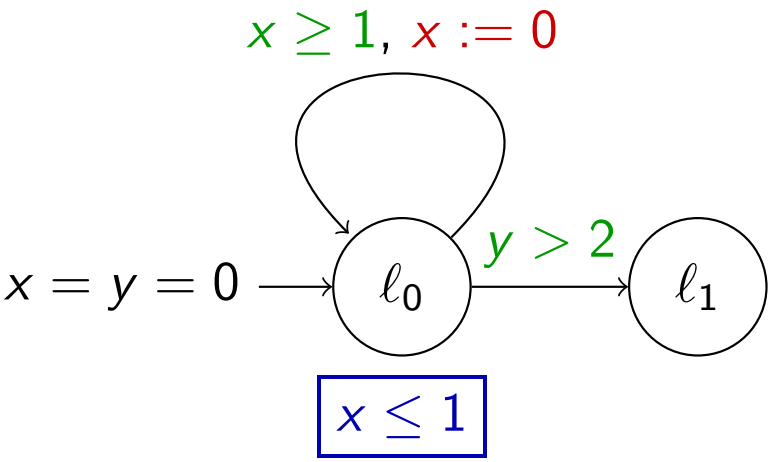

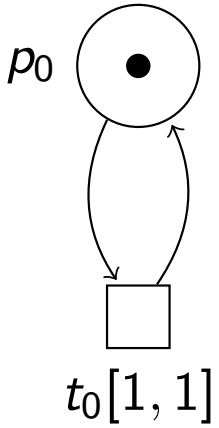

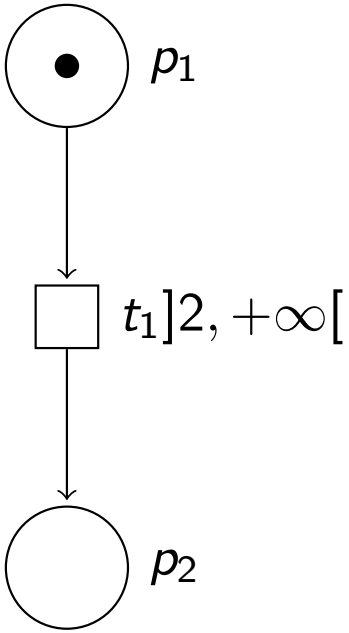

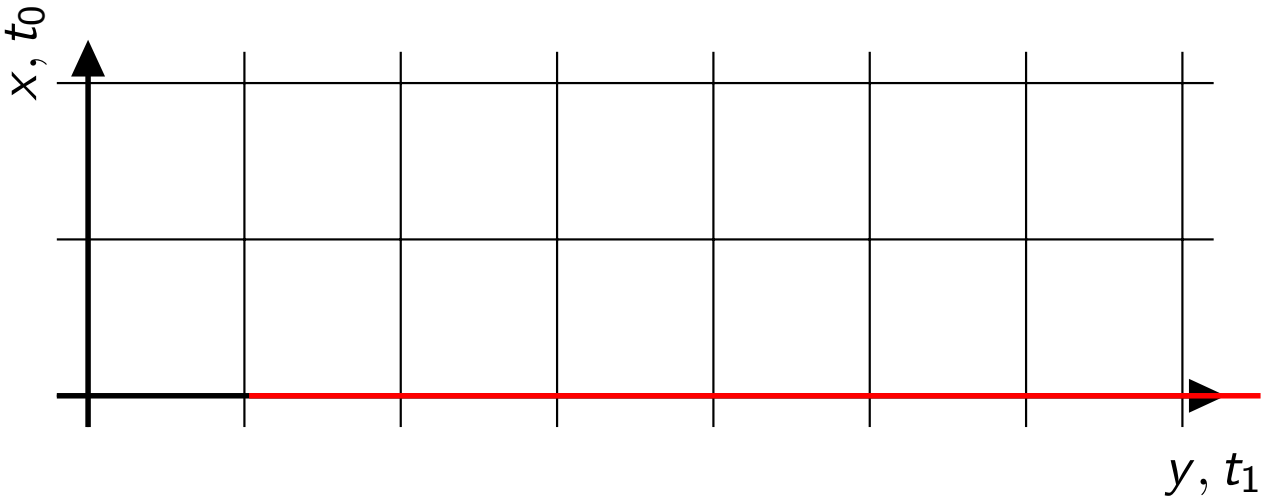

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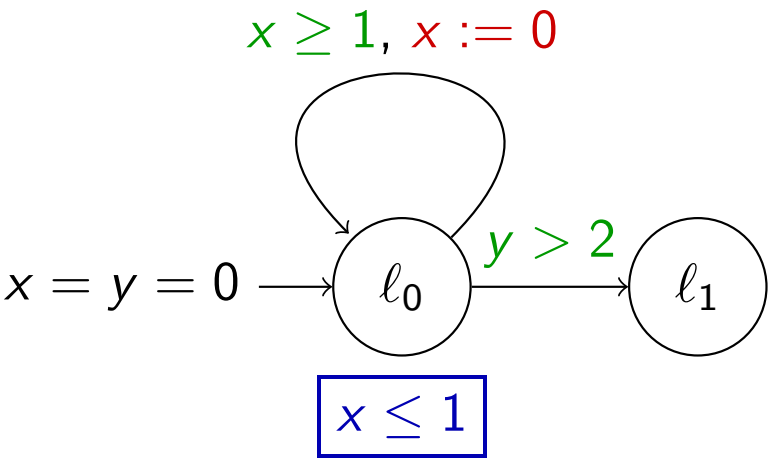

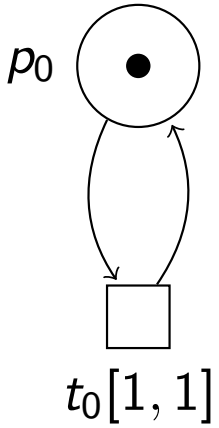

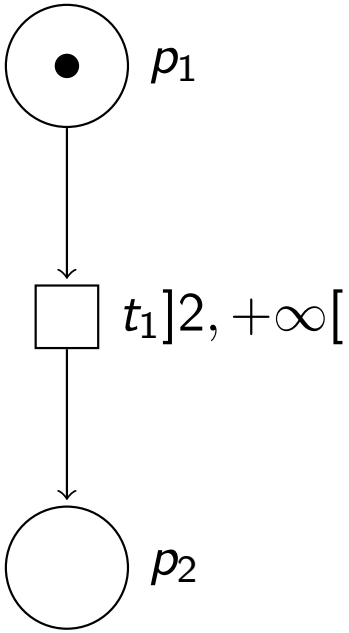

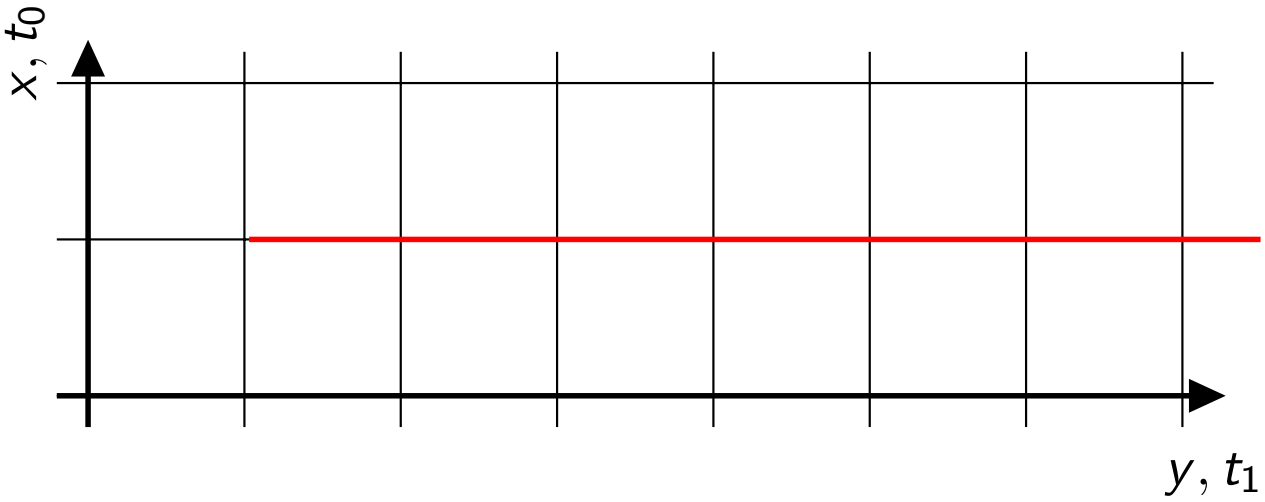

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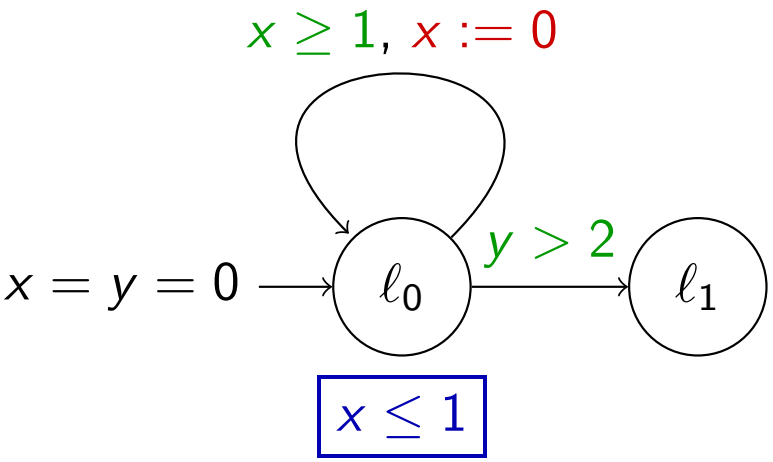

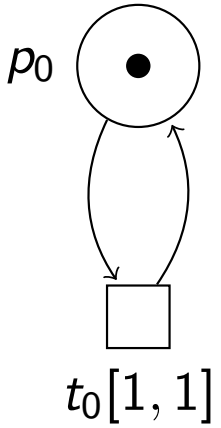

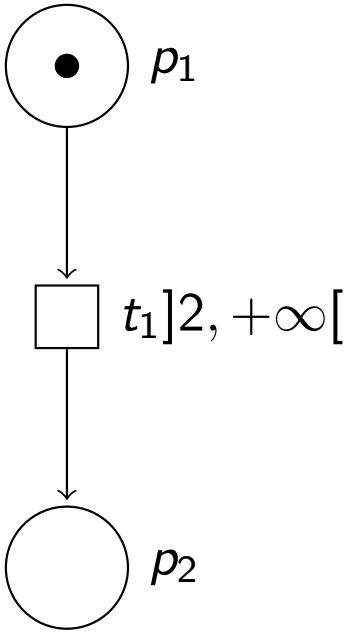

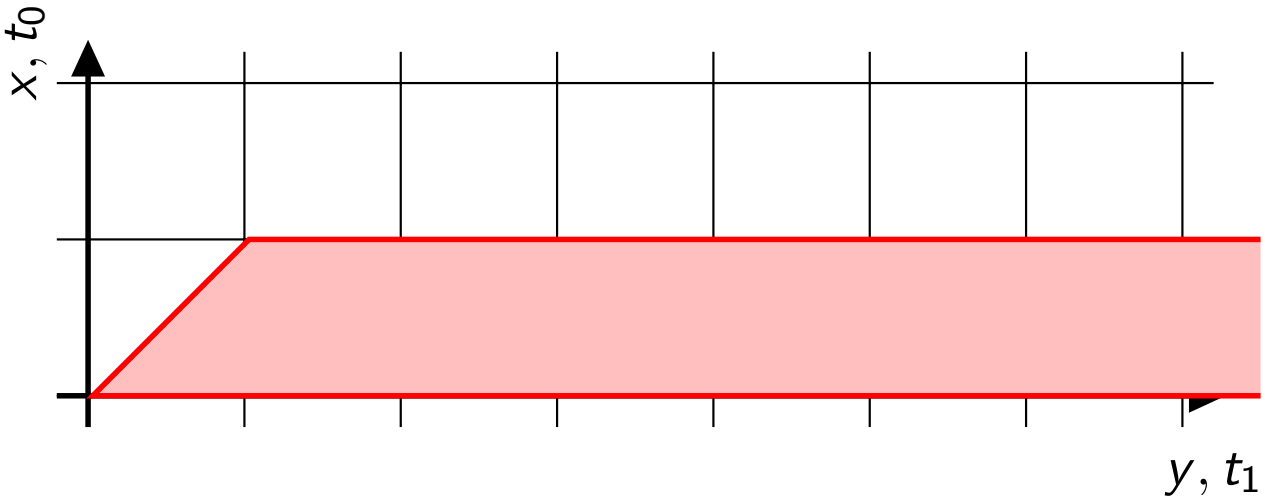

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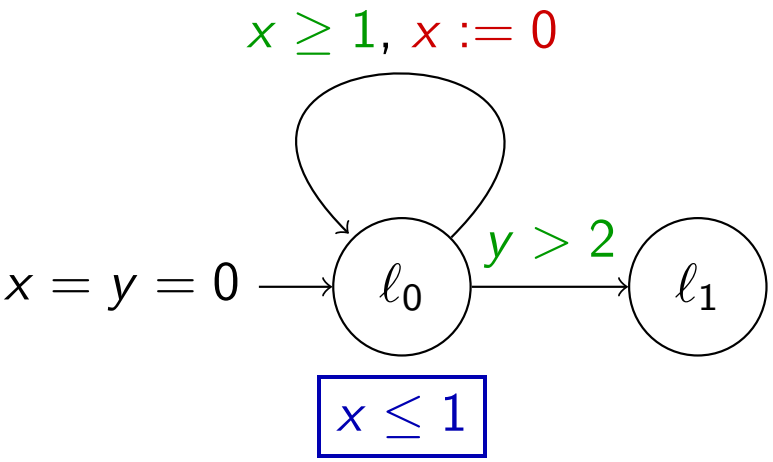

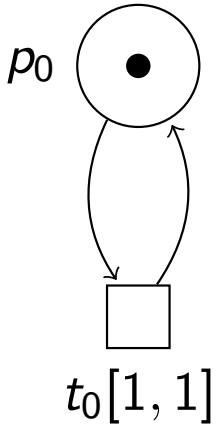

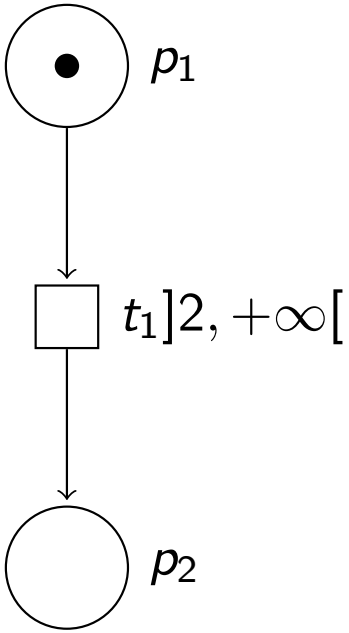

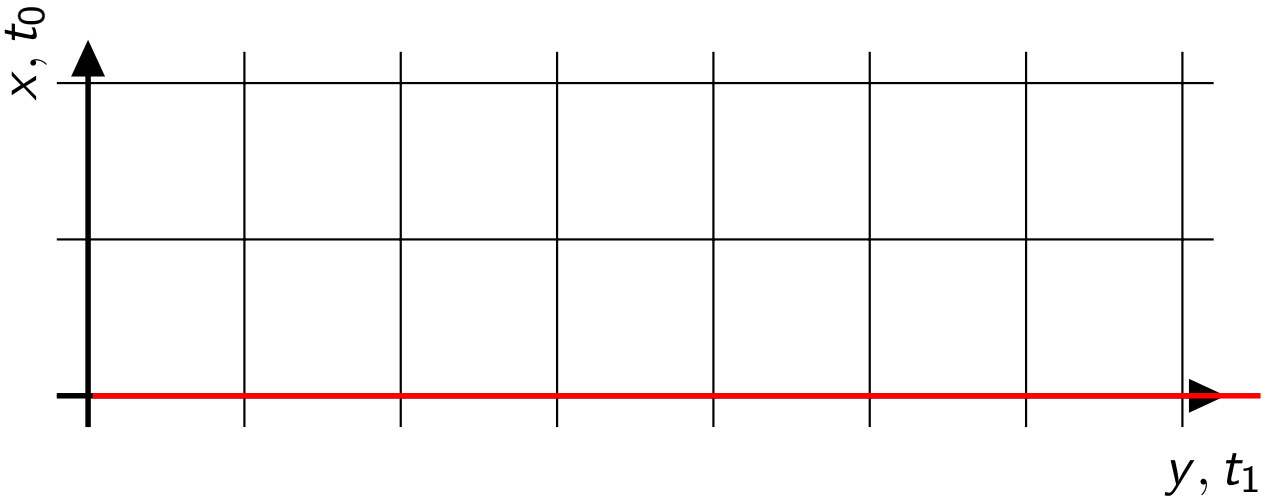

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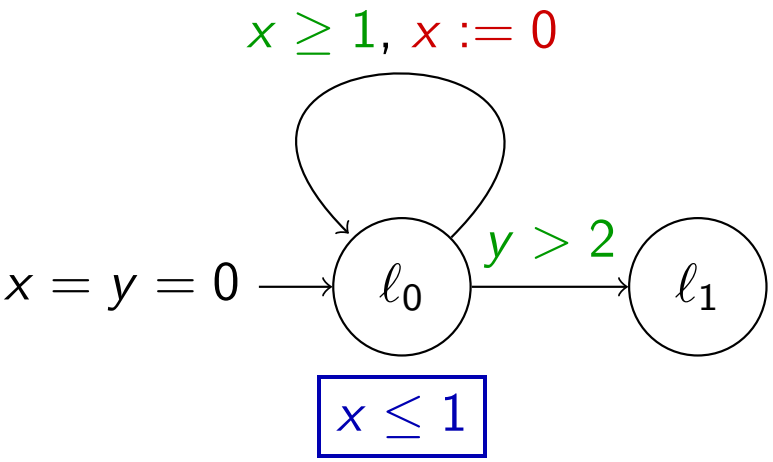

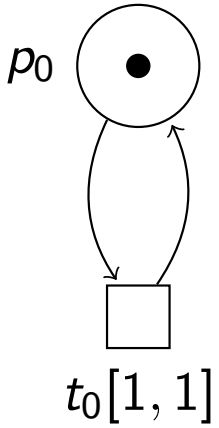

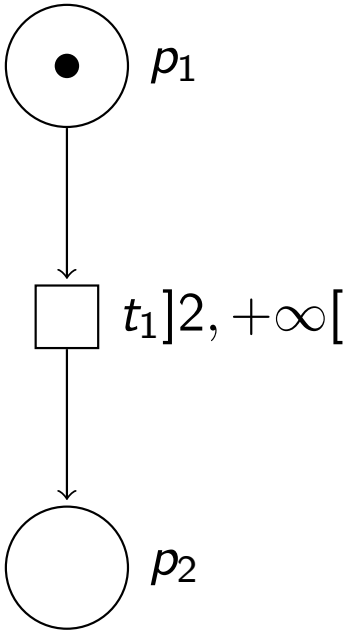

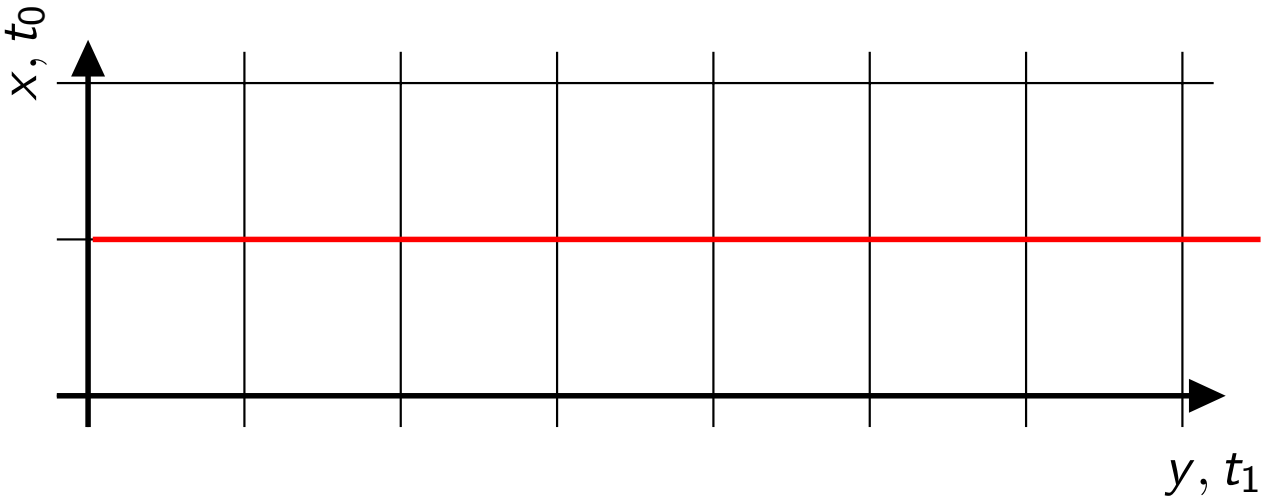

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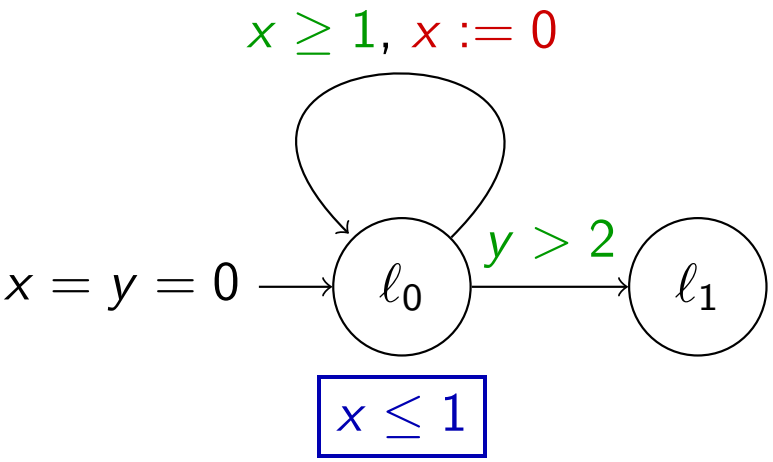

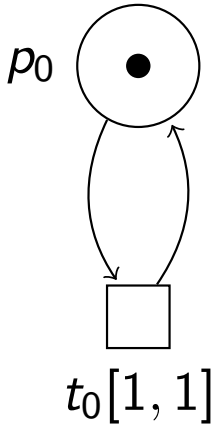

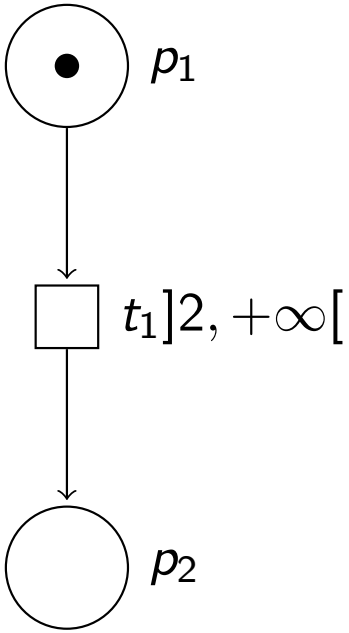

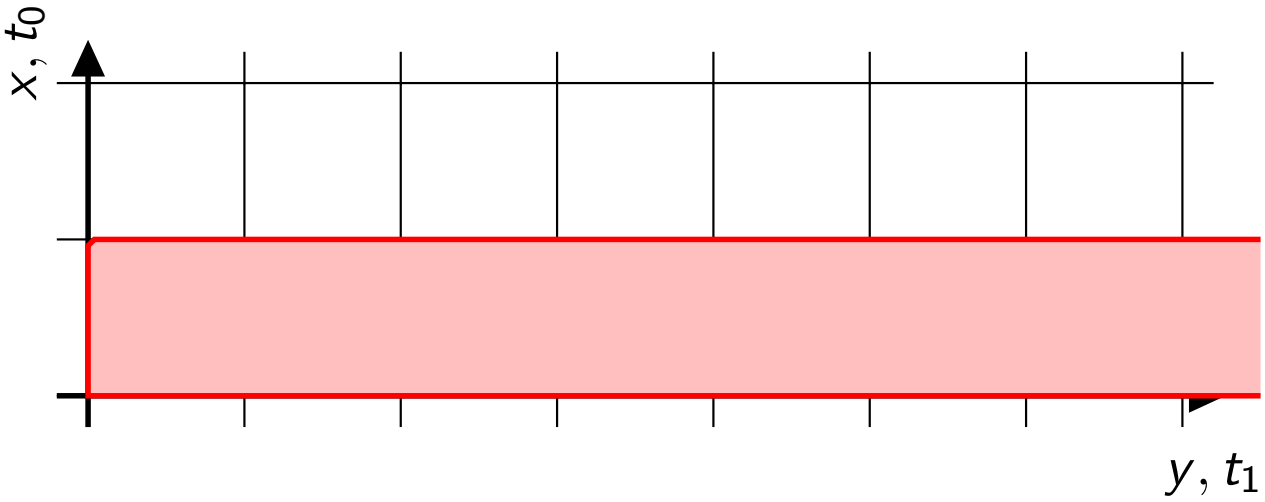

A Scheduling Problem (adapted from [BFSV04])

- Three real-time tasks τ_1, τ_2 and τ_3:
  - τ_1 is periodic with period 50 and execution time $C_1 \in [10, 20]$;
  - τ_2 is sporadic with min. delay 100 and execution time $C_2 \in [18, 28]$;
  - τ_3 is periodic with period 150 and execution time $C_3 \in [20, 28]$;
- Scheduling policy: priority with $\tau_1 > \tau_2 > \tau_3$ (non preemptive);
- Is it **schedulable**? Each task always has at most one instance running

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A Scheduling Problem (adapted from [BFSV04])

- Three real-time tasks τ_1, τ_2 and τ_3:
  - τ_1 is periodic with period a and execution time $C_1 \in [10, 20]$;
  - τ_2 is sporadic with min. delay $2a$ and execution time $C_2 \in [18, 28]$;
  - τ_3 is periodic with period $3a$ and execution time $C_3 \in [20, 28]$;
- Scheduling policy: priority with $\tau_1 > \tau_2 > \tau_3$ (non preemptive);
- Is it **schedulable**? Each task always has at most one instance running

A Scheduling Problem (adapted from [BFSV04])

- Three real-time tasks τ_1, τ_2 and τ_3:
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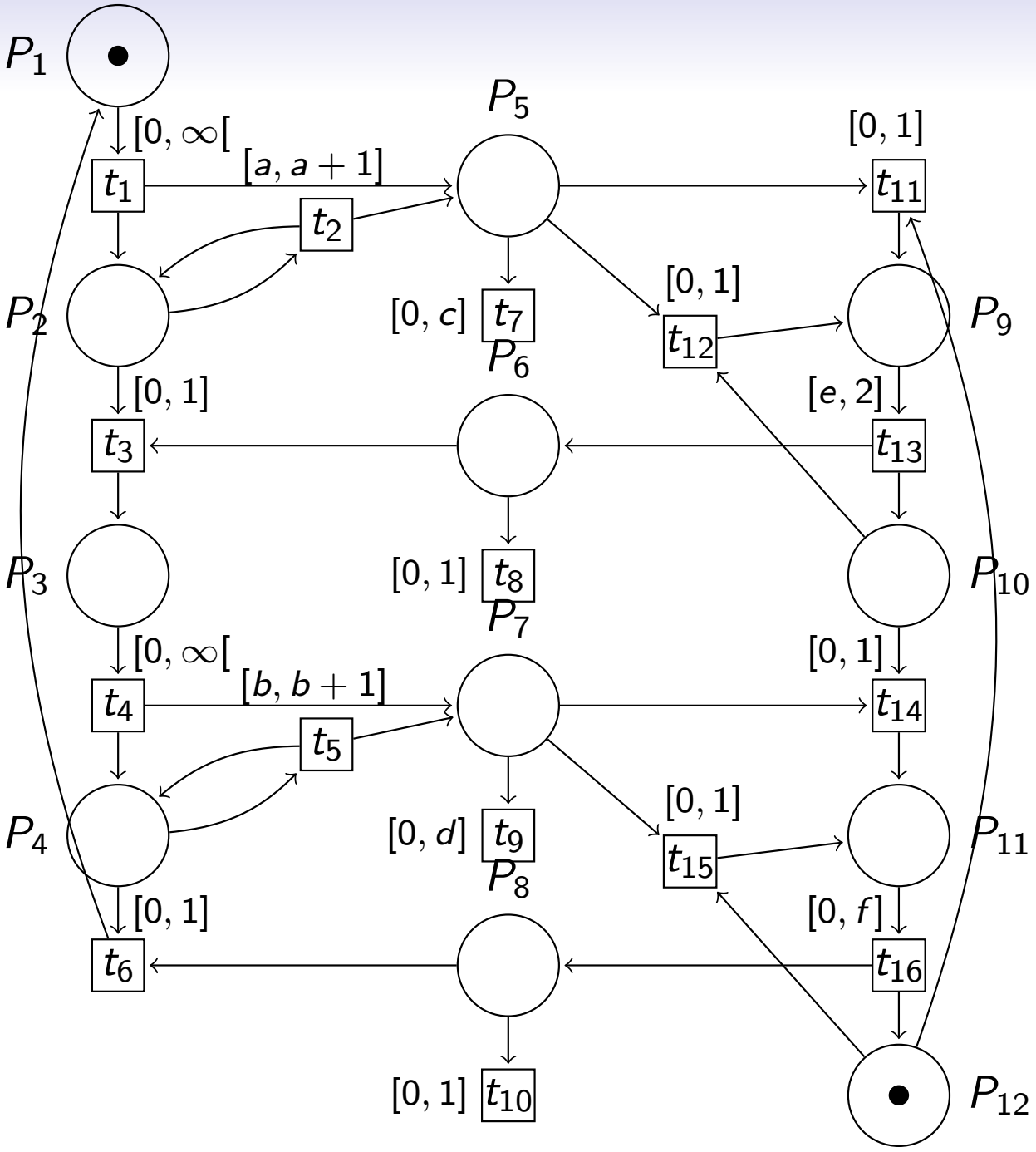
  - τ_3 is periodic with period $3a$ and execution time $C_3 \in [20, 28]$;
- Scheduling policy: priority with $\tau_1 > \tau_2 > \tau_3$ (non preemptive);
- Is it **schedulable**? Each task always has at most one instance running
- What is the minimum value of a such that the system is schedulable?


P_1
$[0, \infty[$
t_1
$[a, a+1]$
t_2
P_5
$[0, 1]$
t_{11}
P_2
$[0, c]$
t_7
P_6
$[0, 1]$
t_{12}
P_9
$[0, 1]$
t_3
$[e, 2]$
t_{13}
P_3
$[0, 1]$
t_8
P_7
P_{10}
$[0, \infty[$
t_4
$[b, b+1]$
t_5
$[0, 1]$
t_{14}
P_4
$[0, d]$
t_9
P_8
$[0, 1]$
t_{15}
P_{11}
$[0, 1]$
t_6
$[0, f]$
t_{16}
$[0, 1]$
t_{10}
P_{12}

For the alternating bit protocol, we have obtained the following constraint with all six parameters as non-negative integers:

$$\begin{cases} c = 0 \\ a \geq 4 \\ e \leq 2 \\ b - f \geq 3 \end{cases} \text{ or } \begin{cases} e \leq 2 \\ b - f \geq 3 \\ a \geq 5 \end{cases} \text{ or } \begin{cases} e \leq 2 \\ -b + d + f \leq -2 \\ a \geq 5 \end{cases}$$

$$\text{or } \begin{cases} e = 0 \\ a - c \geq 4 \\ b - f \geq 3 \end{cases} \text{ or } \begin{cases} e = 0 \\ a - c \geq 4 \\ -b + d + f \leq -2 \end{cases} \text{ or } \begin{cases} c = 0 \\ a \geq 4 \\ e \leq 2 \\ -b + d + f \leq -2 \end{cases}$$

	a, b	a, b, c	a, b, c, d	a, b, c, d, e	a, b, c, d, e, f
EF Time	0 s	0.2 s	0.7 s	DNF	DNF
EF Mem.	1.6 Mo	2.4 Mo	4.7 Mo	DNF	DNF
IEF Time	0 s	0.2 s	0.6 s	44.7 s	152.7 s
IEF Mem.	1.6 Mo	2.5 Mo	4.4 Mo	75.2 Mo	106.9 Mo

Table : Scaling up the number of parameters for the ABP problem

	$a \leq 10$	$a \leq 100$	$a \leq 1000$	$a \leq 10000$
IEF Time	152.8 s	152.6 s	154 s	153.2 s
Int. Hull	2.3 s (%)	2.3 s (%)	2.3 s (%)	2.3 s (%)
IEF Mem.	108.2 Mo	108.2 Mo	108.2 Mo	108.2 Mo

Table : Scaling up a's upper bound for ABP problem

Outline

Summary

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- Decidable model-checking of parametrized timed systems

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- Decidable model-checking of parametrized timed systems
- Control problems: extension to timed games

Conclusion and Perspectives

Conclusion and Perspectives

Bounded integer parameter synthesis:

- explore other timed models (priced timed automata, stopwatch automata,...)

Conclusion and Perspectives

Bounded integer parameter synthesis:

- explore other timed models (priced timed automata, stopwatch automata,...)

Parametric timed games:

- state class graph

Merci de votre attention.

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